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Chapter

1

Electromagnetic Phenomena Not Explained by Maxwell's Equations[260]

Overview

The conventional Maxwell theory is a classical linear theory in which the scalar and vector potentials appear to be arbitrary and defined by boundary conditions and choice of gauge. The conventional wisdom in engineering is that potentials have only mathematical, not physical, significance. However, besides the case of quantum theory, in which it is well known that the potentials are physical constructs, there are a number of physical phenomena — *both classical and quantum-mechanical* — which indicate that the $\mathbf{A}_\mu$ fields, $\mu = 0, 1, 2, 3$, *do* possess physical significance as global-to-local operators or gauge fields, in precisely constrained topologies.

Maxwell's linear theory is of U(1) symmetry form, with Abelian commutation relations. It can be extended to include physically meaningful $\mathbf{A}_\mu$ effects by its reformulation in SU(2) and higher symmetry forms. The commutation relations of the conventional classical Maxwell theory are Abelian. When extended to SU(2) or higher

symmetry forms, Maxwell's theory possesses non-Abelian commutation relations, and addresses global, i.e. nonlocal in space, as well as local phenomena with the potentials used as local-to-global operators.

An adapted Yang–Mills interpretation of *low energy fields* is applied in the following pages — an adaptation previously applied only to *high energy fields*. This adaptation is permitted by precise definition of the topological boundary conditions of those low energy electromagnetic fields. The Wu–Yang interpretation of Maxwell's theory implies the existence of magnetic monopoles and magnetic charge. As the classical fields considered here are low energy fields, *these theoretical constructs are pseudoparticle, or instanton, low energy monopoles and charges*, rather than high energy monopoles and magnetic charge (cf. Refs. 1 and 2).

Although the term "classical Maxwell theory" has a conventional meaning, this meaning actually refers to the interpretations of Maxwell's original writings by Heaviside, Fitzgerald, Lodge and Hertz. These *later interpretations of Maxwell* actually depart in a number of significant ways from Maxwell's original intention. In Maxwell's original formulation, *Faraday's electrotonic state, the A field, was central*, making this prior-to-interpretation, original Maxwell formulation compatible with Yang–Mills theory, and naturally extendable.

The *interpreted classical Maxwell theory* is, as stated, a linear theory of U(1) gauge symmetry. The mathematical dynamic entities called solitons can be either classical or quantum-mechanical, *linear or nonlinear* (cf. Refs. 3 and 4), and describe electromagnetic waves propagating through media. However, solitons are of SU(2) symmetry form.[259] In order for the conventional *interpreted classical Maxwell theory* of U(1) symmetry to describe such entities, the theory must be extended to SU(2) form.

This recent extension of soliton theory to linear equations of motion, together with the recent demonstration that the nonlinear Schrödinger equation and the Korteweg–de-Vries equation — equations of motion with soliton solutions — are reductions of the self-dual Yang–Mills equation (SDYM),[5] are pivotal in understanding the extension of Maxwell's U(1) theory to higher order symmetry forms

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such as SU(2). Instantons are solutions to SDYM equations which have minimum action. The use of Ward's SDYM twistor correspondence for universal integrable systems means that instantons, twistor forms, magnetic monopole constructs and soliton forms all have a pseudoparticle SU(2) correspondence.

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Prolegomena A: Physical Effects Challenging a Maxwell Interpretation

A number of physical effects strongly suggest that the Maxwell field theory of electromagnetism is incomplete. Representing the influence of the independent variable, x, and the dependent variable, y, as $x \rightarrow y$, these effects address: field(s) $\rightarrow$ free electron (F $\rightarrow$ FE), field(s) $\rightarrow$ conducting electron (F $\rightarrow$ CE), field(s) $\rightarrow$ particle (F $\rightarrow$ P), wave guide $\rightarrow$ field (WG $\rightarrow$ F), conducting electron $\rightarrow$ field(s) (CE $\rightarrow$ F) and rotating frame $\rightarrow$ field(s) (RF $\rightarrow$ F) interactions. A nonexhaustive list of these experimentally observed effects, all of which involve the $\mathbf{A}_\mu$ four-potentials (vector and scalar potentials) in a physically effective role, includes:

1. *The Aharonov–Bohm and Altshuler–Aronov–Spivak effects* (F $\rightarrow$ FE and F $\rightarrow$ CE). Ehrenberg and Siday, Aharonov and Bohm, and Altshuler, Aronov and Spivak predicted experimental results by attributing physical effects to the $\mathbf{A}_\mu$ potentials. Most commentaries in classical field theory still show these potentials as mathematical conveniences without gauge invariance and with no physical significance.
2. *The topological phase effects of Berry, Aharonov, Anandan, Pancharatnam, Chiao and Wu* (WG $\rightarrow$ F and F $\rightarrow$ P). In the WG $\rightarrow$ F version, the polarization of light is changed by changing the spatial trajectory adiabatically. The Berry–Aharonov–Anandan phase has also been demonstrated at the quantum as well as the classical level. This phase effect in parameter (momentum) space is the correlate of the Aharonov–Bohm effect in metric (ordinary) space, both involving adiabatic transport.

3. *The Josephson effect* (CE → F). At both the quantum and the macrophysical level, the free energy of the barrier is defined with respect to an $\mathbf{A}_\mu$ potential variable (phase).
4. *The quantum Hall effect* (F → CE). Gauge invariance of the $\mathbf{A}_\mu$ vector potential, being an exact symmetry, forces the addition of a flux quantum to result in an excitation without dependence on the electron density.
5. *The De Haas–Van Alphen effect* (F → CE). The periodicity of oscillations in this effect is determined by $\mathbf{A}_\mu$ potential dependency and gauge invariance.
6. *The Sagnac effect* (RF → F). Exhibited in the well-known and well-used ring laser gyro, this effect demonstrates that the Maxwell theory, as presently formulated, does not make explicit the constitutive relations of free space, and does not have a built-in Lorentz invariance as its field equations are independent of the metric.

The $\mathbf{A}_\mu$ potentials have been demonstrated to be physically meaningful constructs at the quantum level (effects 1–5), at the classical level (effects 2, 3 and 6), and at a relatively long range in the case of effect 2. In the F → CE and CE → F cases (effects 1, 3–5), the effect is limited by the temperature-dependent electron coherence length with respect to the device/antenna length.

The Wu–Yang theory attempted the completion of Maxwell's theory of electromagnetism by the introduction of a nonintegrable (path-dependent) phase factor (NIP) as a physically meaningful quantity. The introduction of this construct permitted the demonstration of $\mathbf{A}_\mu$ potential gauge invariance and gave an explanation of the Aharonov–Bohm effect. The NIP is implied by the magnetic monopole and magnetic charge theoretical constructs viewed as *pseudoparticles* or *instantons*.[a]

The recently formulated Harmuth ansatz also addresses the incompleteness of Maxwell's theory: an amended version of Maxwell's

[a]The term "instanton" or "pseudoparticle" is defined as the minimum action solutions of SU(2) Yang–Mills fields in Euclidean four-space R^4.[32]

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equations can be used to calculate e.m. signal velocities provided that (a) a magnetic current density and (b) a magnetic monopole theoretical construct are assumed.

Formerly, treatment of the $\mathbf{A}_\mu$ potentials as anything more than mathematical conveniences was prevented by their obvious lack of gauge invariance.[251,252] However, gauge invariance for the $\mathbf{A}_\mu$ potentials results from situations in which fields, firstly, have a history of separate spatiotemporal conditioning and, secondly, are mapped in a many-to-one, or global-to-local, fashion (in holonomy). Such conditions are satisfied by $\mathbf{A}_\mu$ potentials with boundary conditions, i.e. the usual empirically encountered situation. Thus, with the correct geometry and topology (i.e. with stated boundary conditions) the $\mathbf{A}_\mu$ potentials always have physical meaning. This indicates that Maxwell's theory can be extended by the appropriate use of topological and gauge-symmetrical concepts.

The $\mathbf{A}_\mu$ potentials are local operators mapping global spatiotemporal conditions onto the local e.m. fields. The effect of this operation is measurable as a phase change, if there is a second, comparative mapping of differentially conditioned fields in a many-to-one (global-to-local) summation. With coherent fields, the possibility of measurement (detection) after the second mapping is maximized.

The conventional Maxwell theory is incomplete due to the neglect of (1) a definition of the $\mathbf{A}_\mu$ potentials as operators on the local intensity fields dependent on gauge, topology, geometry and global boundary conditions; and of (2) a definition of the constitutive relations between medium-independent fields and the topology of the medium.[b] Addressing these issues extends the conventional Maxwell theory to cover physical phenomena which cannot be presently explained by that theory.

[b]The paper by Konopinski[253] provides a notable exception to the general lack of appreciation of the central role of the **A** potentials in electromagnetism. Konopinski shows that the equations from which the Lorentz potentials $\mathbf{A}_\nu$ $(\mathbf{A}, \phi)$ arising from the sources j_n $(\mathbf{j}, \rho)$ are derived are $\partial_\mu^2 \mathbf{A}_\nu = -4\pi \mathrm{j}_\nu / c$, $\partial_\mu \mathbf{A}_\mu = 0$, and can displace the Maxwell equations as the basis of electromagnetic theory. The Maxwell equations follow from these equations whenever the antisymmetric field tensor $F_{\mu\nu}(\mathbf{E}, \mathbf{B}) = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$ is defined.

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Prolegomena B: Interpretation of Maxwell's Original Formulation

B.1. *The Faraday–Maxwell formulation*

Central to the Maxwell formulation of electromagnetism[6–16] was the Faraday concept of the *electrotonic state* (from the new Latin *tonicus*, "of tension or tone"; from the Greek *tonos*, "a stretching"). Maxwell defined this state as the "fundamental quantity in the theory of electromagnetism" on the *changes of which* (not on its absolute magnitude) the induction current depends (Ref. 8: vol. 2, p. 540). Faraday had clearly indicated the fundamental role of this state in his two circuit experiments (Ref. 17: vol. 1, series I, 60); and Maxwell endorsed its importance: "The scientific value of Faraday's conception of an electrotonic state consists in its directing the mind to lay hold of a certain quantity, on the changes of which the actual phenomena depend." (Ref. 8, vol. 2, p. 541).

The continental European views of the time concerning propagation (e.g. those of Weber, Biot, Savart and Neumann), which Maxwell opposed, were based on the concept of action-at-a-distance. In place of action-at-a-distance, Maxwell offered *a medium characterized by polarization and strain* through which radiation propagated from one local region to another local region. Furthermore, instead of force residing in the medium, Maxwell adopted another Faraday concept: force fields, or *magnetic lines of force independent of matter or magnet*. In contrast, Weber's position was that force is dependent on relative velocity and acceleration. However, Maxwell's response was not made in rebuttal of Weber's. Rather, Maxwell believed that there are two ways of looking at the subject[14]: his own and Weber's. Certainly, Helmholtz achieved a form of synthesis of the two pictures.[18]

For Maxwell, the distinction between *quantity* and *intensity* was also central. For example, magnetic intensity was represented by a line integral and referred to the magnetic polarization of the medium. Magnetic quantity was represented by a surface integral and referred to the magnetic induction in the medium. In all cases, a medium was required and *the medium was the seat of electromagnetic phenomena*. The electromagnetic and luminiferous medium were identified.

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Faraday's lines of force were said to indicate the direction of minimum pressure at every point in the medium and constituted the field concept. Thus, for Maxwell, the electromagnetic field did not exist, *sui generis*, but as a state of the medium, and the mechanical cause of differences in pressure in the medium was accounted for by the hypothesis of vortices, i.e. polarization vectors. The medium was also restricted to be one in which there was only a displacement current (with no conduction currents) and in which there were no electrical or magnetic sources. Furthermore, rather than electricity *producing* a disturbance in the medium (which was W. Thomson's view), Maxwell's field theory described the presence of electricity as a disturbance, i.e. the electricity *was* the disturbance.

Maxwell conceived of electric current as a moving system with forces communicating the motion from one part of the system to another. The nature of the forces was undefined, and did not need to be defined. As for Maxwell, the forces could be eliminated from the equations of motion by Lagrangian methods for any connected system. That is, *the equations of motion were defined only locally*. Thus, Maxwell dispensed with the dynamic forces permitting propagation through the medium; only the beginning and end of the propagating process was examined and that only locally. Therefore, only the local state of the medium, or the electrotonic intensity or state, was primary, and that corresponds to Maxwell's "F" or "α_o" — or, in modern symbols, the **A** field. This is where matters stood until 1873, with the vector potential playing a pivotal *physical* role in Maxwell's theory.

B.2. *The British Maxwellians and the Maxwell–Heaviside formulation*

Subsequently, the **A** field was *banished from playing the central role in Maxwell's theory and relegated to being a mathematical* (*but not physical*) *auxiliary*. This banishment took place during the interpretation of Maxwell's theory by the Maxwellians,[12] i.e. chiefly by Heaviside, Fitzgerald, Lodge and Hertz. The "Maxwell theory" and "Maxwell's equations" we know today are really the interpretation of

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Maxwell by these Maxwellians.[12,16] It was Heaviside who "murdered the **A** field" (Heaviside's description) and whose work influenced the crucial discussion which took place at the 1888 Bath meeting of the British Association (although Heaviside was not present). The "Maxwell's equations" of today are due to Heaviside's "redressing" of Maxwell's work, and should, more accurately, be known as the "Maxwell–Heaviside equations." Essentially, Heaviside took the twenty equations of Maxwell and reduced them to the four now known as "Maxwell's equations."

The British Maxwellians, Heaviside, Fitzgerald and Lodge, may have banished scalar and vector potentials from the propagation equations, but the center of concern for them remained the *dynamic state of the medium, or ether*. The banishment of the potentials can today be justified in the light of the discussion to follow in that the Maxwell theory focused on local phenomena, and the **A** field, as we shall see, addresses global connectivity. Therefore, in order for the theory to progress, it was perhaps better that the **A** field was put aside, or at least assigned an auxiliary role, *at that time*. Heaviside's comment that the electrostatic potential was a "physical inanity" was probably correct for the 19th century but, as will be shown below, the potential regained its sanity in the 20th century — starting with the work of Hermann Weyl.

But it should be emphasized that the British Maxwellians retained the focus of theory *on the medium*. Both Heaviside and Poynting agreed that the function of a wire is as a sink into which energy passes from the medium (ether) and is convected into heat. For them, wires conduct electricity with the Poynting vector pointing at right angles to the conducting wire (cf. Ref. 19, Sec. 27-5). The modern conventional view on conduction in wires is similar, but modern theory is not straightforward about where this energy goes, yet still retains Poynting's theorem. The energy flows, not through a current-carrying wire itself, but through the medium (ether) around it — or, rather, through whatever energy-storing substance a modern theorist imagines exists in the absence of the ether. Nonetheless, Heaviside was probably correct to banish scalar and vector potentials from propagation equations due to the fact that the notion of gauge invariance

(*Mass-stabinvarianz* — see below) was not yet conceived and thus not known to the Maxwellians. However, these **A** fields still remained as a repository of energy in the electrotonic state of the medium and the redressed and interpreted Maxwell theory of the British Maxwellians remained a *true dynamic* theory of electromagnetism.

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B.3. *The Hertzian and current classical formulation*

But all dynamics were banished by Hertz. Hertz banished even the stresses and strains of the medium (ether) and was vigorously opposed in this by the British Maxwellians.[12] Hertz even went far beyond his mentor, Helmholtz, in his austere operational formulation. Nonetheless, the Hertz orientation finally prevailed, and the modern "Maxwell theory" is today a system of equations describing electrodynamics *which has lost its dynamical basis*.

Another significant reinterpretation of Maxwell took place in which Heaviside was involved. The 19th century battle between Heaviside and Tait concerning the use of quaternions and culminating in the victory of Heaviside and vector analysis may also be reassessed in the light of modern developments. *Without* the concepts of gauge, global (as opposed to local) fields, nonintegrable phase factors (see below) and topological connections, the use of quaternions was getting in the way of progress. That is not to say that either quaternionic algebra or the potentials were, or are, unphysical or unimportant. It is to say, rather, that the potentials could not be understood *then* with the limited theory and mathematical tools available *then*. Certainly it is *now* realized that the algebraic formulation of electromagnetism is more complicated than to be described completely even by quaternionic algebra, and certainly more complicated than to be described by simple vector analysis (cf. Ref. 20).

But to return to Maxwell's original formulation: Maxwell *did* place the **A** field center stage and *did* use quaternionic algebra to dress his theory. We know now that quaternionic algebra is described by the SU(2) group of transformations, and vector algebra by the U(1) group of transformations. As such modern propagation phenomena such as solitons are of SU(2) form, we might even view the original Maxwell

formulation as more comprehensive than that offered by the British Maxwellian interpretation, and certainly more of a dynamic theory than the physically unintuitive local theory finally adopted by Hertz. That said, when it is stated, below, that the "Maxwell equations" need extension, it is really the modern Heaviside–Hertz interpretation that is meant. The *original* Maxwell theory could have been easily extended into Yang–Mills form.

Between the time of Hertz's interpretation of Maxwell's theory and the appearance of the gauge field concepts of Hermann Weyl, there appeared Whittaker's notable *mathematical* statements[21,22] that (i) the force potential can be defined in terms of both standing waves and propagating waves and (ii) any electromagnetic field e.g. dielectric displacement and magnetic force, can be expressed in terms of the derivatives of two scalar potential functions, and also be related to an inverse square law of attraction.

Whittaker[21] commenced his statement with Laplace's equation:

$$\frac{\partial^2 V}{dx^2} + \frac{\partial^2 V}{dy^2} + \frac{\partial^2 V}{dz^2} = 0, \tag{P.1}$$

which is satisfied by the potential of any distribution of matter which attracts according to Newton's law. The potential at any point (x, y, z) of any distribution of matter of mass m, situated at the point (a, b, c) which attracts according to this law, is

$$\int_0^{2\pi} f(z + ix\cos u + iy\sin u, u)du, \tag{P.2}$$

where u is a periodic argument. The most general solution to Laplace's equation using this expression is

$$\mathrm{V} = \int_0^{2\pi} f(z + ix\cos u + iy\sin u, u)du, \tag{P.3}$$

where f is an arbitrary function of the two arguments:

$$z + ix\cos u + iy\sin u \quad \text{and} \quad u. \tag{P.4}$$

In order to express this solution as a series of harmonic terms, Whittaker[21] showed that it is only necessary to expand the function

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f as a *Taylor series* with respect to the first argument $z + ix\cos u + iy \sin u$, and as a *Fourier series* with respect to the second argument, u.

Whittaker also showed that the general solution to the partial differential wave equation,

$$\frac{\partial^2 V}{dx^2} + \frac{\partial^2 V}{dy^2} + \frac{\partial^2 V}{dz^2} = \frac{k^2 \partial^2 V}{dt^2}, \tag{P.5}$$

is

$$\mathrm{V} = \int_0^{2\pi} \int_0^{\pi} f\left(x \sin u \cos v + y \sin u \sin v + z \cos u + \frac{t}{k}, u, v\right) du dv, \tag{P.6}$$

where f is now an arbitrary function of the three arguments:

$$x \sin u \cos u + y \sin u \sin v + z \cos u + \frac{t}{k}, u \quad \text{and} \quad v, \tag{P.7}$$

and can be analyzed into a simple plane wave solution. Therefore, for any force varying as the inverse square of the distance, the potential of such a force satisfies *both* Laplace's equation *and* the wave equation, and can be analyzed into simple plane waves propagating with constant velocity. The *sum of these waves*, however, does not vary with time, i.e. *they are standing waves*. Therefore, the force potential can be defined in terms of both standing waves, i.e. by a global, or nonlocal, solution, and by propagating waves, i.e. by a local solution changing in time.

Furthermore, Whittaker[22] demonstrated that any electromagnetic field, e.g. dielectric displacement and magnetic force, can be expressed in terms of the derivatives of *two* scalar potential functions, F and G, satisfying

$$\begin{aligned} \frac{\partial^2 F}{dx^2} + \frac{\partial^2 F}{dy^2} + \frac{\partial^2 F}{dz^2} - \left[\frac{1}{c^2}\right] \frac{\partial^2 F}{dt^2} = 0, \\ \frac{\partial^2 G}{dx^2} + \frac{\partial^2 G}{dy^2} + \frac{\partial^2 G}{dz^2} - \left[\frac{1}{c^2}\right] \frac{\partial^2 G}{dt^2} = 0. \end{aligned} \tag{P.8}$$

Thus, Whittaker's mathematical statement related the inverse square law of force to the force potential defined in terms of both standing wave (i.e. global) and propagating wave (i.e. local) solutions. The analysis also showed that the electromagnetic force fields could be defined

in terms of the derivatives of two scalar potentials. This was the state of affairs prior to Weyl's introduction of gauge fields.

The landmark work of Weyl[23–26] and Yang and Mills[27] has been matched by the conception of pseudoparticle or minimum action solutions to the Yang–Mills equations,[1] i.e. instantons. Such phenomena and the appearance of gauge structure are found in simple dynamical, or classical, systems,[28] and the concept of instanton has been the focus of intense activity in recent years (cf. Refs. 2, 29–33).

The demonstration that the nonlinear Schrödinger equation and the Korteweg–de-Vries equation — equations with soliton solutions — are reductions of the self-dual Yang–Mills equations[5] with correspondences to twistor formulations[34] has provided additional evidence concerning the direction that Maxwell's theory must take. These reductions of self-dual Yang–Mills equations are known to apply to various classical systems, depending on the choice of Lie algebra associated with the self-dual fields.[35]

It is also relevant that the soliton mathematical concept need not result only from nonlinear equations. Recently, Barut[3] and Shaaarawi and Besieris[4] have demonstrated that soliton solutions are possible in the case of *linear* de-Broglie-like wave equations. Localized oscillating finite energy solutions to the massless wave equation are derived which move like massive relativistic particles with energy $E = \lambda\omega$ and momentum $\mathbf{p} = \lambda\mathbf{k}$ (λ = const.). Such soliton solutions to linear wave equations do not spread and have a finite energy field.

1. Introduction

There are a number of reasons for questioning the completeness of the conventionally interpreted Maxwell theory of electromagnetism. It is well known that there is an arbitrariness in the definition of the **A** vector and scalar potentials, which, nevertheless, have been found very useful when used in calculations with boundary conditions known.[253] The reasons for questioning completeness are due to experimental evidence (Sec. 3), theoretical (Sec. 4) and pragmatic (Sec. 5).

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An examination of the Maxwell theory may begin with the well-known Maxwell equations[c]:

Coulomb's law:

$$\nabla \cdot \mathbf{D} = 4\pi\rho; \tag{1.1}$$

Maxwell's generalization of Ampere's Law:

$$\nabla \times \mathbf{H} = \left(\frac{4\pi}{c}\right)\mathbf{J} + \left(\frac{1}{c}\right)\frac{\partial \mathbf{D}}{\partial t}; \tag{1.2}$$

the postulate of an absence of free *local* magnetic poles or the differential form of Gauss' law:

$$\nabla \cdot \mathbf{B} = 0; \tag{1.3}$$

and Faraday's Law:

$$\nabla \times \mathbf{E} + \left(\frac{1}{c}\right)\frac{\partial \mathbf{B}}{\partial t} = 0. \tag{1.4}$$

The constitutive relations of the medium-independent fields to matter are well known to be

$$\mathbf{D} = \varepsilon\mathbf{E}, \tag{1.5}$$

$$\mathbf{J} = \sigma\mathbf{E}, \tag{1.6}$$

$$\mathbf{B} = \mu\mathbf{H}. \tag{1.7}$$

Because of the postulate of an absence of free local magnetic monopoles [Eq. (1.3)], the following relation is permitted:

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{1.8}$$

but the vector potential, **A**, is thus always arbitrarily defined, because the gradient of some scalar function, L, can be added leaving **B** unchanged, i.e. **B** is unchanged by the gauge transformations:

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\Lambda; \quad \Phi \to \Phi' = \Phi - \left(\frac{1}{c}\right)\frac{\partial \Lambda}{\partial t}. \tag{1.9}$$

This arbitrary definition of the potentials means that *any* gauge chosen is arbitrary, or, an appeal must be made to boundary conditions for any choice.

[c]The equations are in Gaussian or cgs units: centimeter, gram and second. The Système International (SI) or mks units are: meter, kilogram and second.

Now Eq. (1.8) permits a redefinition of Eq. (1.4):

$$\nabla \times \left(\mathbf{E} + \left(\frac{1}{c} \right) \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \quad \text{(Faraday's law rewritten)}, \tag{1.10}$$

which means that the quantity in brackets is the gradient of a scalar function, Φ, and so

$$\mathbf{E} + \left(\frac{1}{c} \right) \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi, \quad \text{or} \quad \mathbf{E} = -\partial \Phi - \left(\frac{1}{c} \right) \frac{\partial \mathbf{A}}{\partial t}, \tag{1.11}$$

and the Maxwell equations (1.3) and (1.4) can be redefined by the use of Eqs. (1.8) and (1.11).

The Maxwell equations (1.1) and (1.2) can also be written as

$$\partial^2 \Phi + \left(\frac{1}{c} \right) \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} = -4\pi\rho, \tag{1.12}$$

$$\nabla^2 \mathbf{A} - \left(\frac{1}{c^2} \right) \left(\frac{\partial^2 \mathbf{A}}{\partial t^2} \right) - \nabla \left(\nabla \cdot \mathbf{A} + \left(\frac{1}{c} \right) \frac{\partial \Phi}{\partial t} \right) = - \left(\frac{4\pi}{c} \right) \mathbf{J}. \tag{1.13}$$

Since the gauge conditions (1.9) are arbitrary, a set of potentials ($\mathbf{A}$, Φ) can be chosen so that

$$\nabla \cdot \mathbf{A} + \left(\left(\frac{1}{c} \right) \frac{\partial \Phi}{\partial t} \right) = 0. \tag{1.14}$$

This choice is called the Lorentz condition or the Lorentz gauge. Equations (1.12) and (1.13) can then be decoupled to obtain

$$\nabla^2 \Phi + \left(\frac{1}{c^2} \right) \frac{\partial^2 \Phi}{\partial t^2} = -4\pi\rho, \tag{1.15}$$

$$\nabla^2 \mathbf{A} - \left(\frac{1}{c^2} \right) \frac{\partial^2 \mathbf{A}}{\partial t^2} = - \left(\frac{4\pi}{c} \right) \mathbf{J}, \tag{1.16}$$

which is useful, because the Maxwell equations are then independent of the coordinate system chosen. Nonetheless, as $\mathbf{A}$ and Φ are not gauge-invariant, the original choice of the Lorentz gauge is arbitrary — a choice which is not an inevitable consequence of the Maxwell

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theory — and the resultant from that choice, namely Eqs. (1.15) and (1.16), is equally arbitrary.[d]

Then again, the arbitrariness of Eq. (1.9) is useful because it permits the choice

$$\nabla \cdot \mathbf{A} = 0. \tag{1.17}$$

Equation (1.12), which is the Maxwell equation (1.1), then permits

$$\nabla^2 \Phi = -4\pi\rho, \tag{1.18}$$

which is the instantaneous Coulomb potential, and hence the condition (1.17) is called the Coulomb or transverse gauge, because the wave equation for **A** can be expressed in terms of the transverse current:

$$\nabla^2 \mathbf{A} - \left(\frac{1}{c^2}\right) \frac{\partial^2 A}{\partial t^2} = -\left(\frac{4\pi}{c}\right) \mathbf{J}_t, \tag{1.19}$$

where $\mathbf{J}_t = \mathbf{J} - \mathbf{J}_l$, with $\mathbf{J}_l$ being the longitudinal current. This is a useful thing to do when no sources are present, but, again, as **A** and Φ are not gauge-invariant, i.e. considered to have no physical meaning, the original choice of the Coulomb gauge is arbitrary, and so is the resultant from that choice, namely Eq. (1.19).

For all that, the absence of gauge invariance (physical meaning) of the **A** vector potential and the Φ scalar potential may seem a fortunate circumstance to those using the Maxwell theory to calculate predictions. These potentials have long been considered a fortunate mathematical convenience, but *just* a mathematical convenience, with no physical meaning. These constructs lack gauge invariance, a defining characteristic of physical, rather than merely mathematical, constructs. What then is meant by a gauge and gauge invariance?

[d]The above account applies to the Hertzian potential and the Hertz vector which are related to **A** and ϕ. However, the Hertzian vector obeys an inhomogeneous wave equation with the polarization vector as source, whereas **A** and ϕ obey their respective wave equations with electric current and charge as source. Furthermore, the Hertzian potential is a three-component potential, whereas **A** and ϕ amount to a four-potential (cf. Ref. 252, p. 254).

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2. What is a Gauge?

In 1918 Weyl[23] (see also Ref. 36) treated Einstein's general theory of relativity as if the Lorentz symmetry were an example of *global* symmetry but with only *local* coordinates defineable, i.e. the general theory was considered as a *local* theory. A consequence of Weyl's theory is that the absolute magnitude or norm of a physical vector is not treated as an absolute quantity but depends on its location in space–time. This notion was called scale (*Mass-stab*) or gauge invariance.

This concept can be understood as follows. Consider a vector at position x with norm given by $f(x)$. If the coordinates are transformed, so that the vector is now at $x + dx$, the norm is $f(x + dx)$. Using the abbreviation ∂/∂^μ, $\mu = 0, 1, 2, 3$, expanding to first order, and using Einstein's summation convention,

$$f(x + dx) = f(x) + \partial_\mu f dx^\mu. \tag{2.1}$$

If a gauge change is introduced by a multiplicative scaling factor, $S(x)$, which equals unity at x, then

$$S(x + dx) = 1 + \partial_\mu S\, dx^\mu. \tag{2.2}$$

If a vector is to be constant under change of location, then

$$Sf = f + [\partial_\mu\, S] f dx^\mu + [\partial_\mu f] dx^\mu \tag{2.3}$$

and, on moving, the norm changes by the amount

$$[\partial_\mu + \partial_\mu S] f dx^\mu. \tag{2.4}$$

Weyl identified $\partial_\mu S$ with the electromagnetic potential $\mathbf{A}_\mu$.

However, this suggestion was rejected (by Einstein) because the natural scale for matter is the Compton wavelength, λ, and as the wave description of matter is $\lambda = \hbar/mc\lambda$ ($\hbar$ is Planck's constant and c is the speed of light), then if, as is always assumed, the wavelength is determined by the particle's mass, m, and with $\hbar$ and c constant (according to the special theory of relativity), λ cannot depend on position without violating the special theory. When made aware of this reasoning, Weyl abandoned his proposal. So the term "gauge change" originally meant "change in length," and was withdrawn from consideration for this particular metric connotation shortly after its introduction.

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But the term did not die. "Gauge invariance" managed to survive in classical mechanics because, with the potentials arbitrary, Maxwell's equations for the **E**, **B**, **H** and **D** fields have a built-in symmetry and such arbitrary potentials became a useful mathematical device for simplifying many calculations in electrodynamics, as we have seen. Nevertheless, the gauge invariance in electromagnetism for the **E**, **B**, **H** and **D** fields was regarded as only an "accidental" symmetry, and the lack of gauge invariance of the electromagnetic vector and scalar potentials was interpreted as an example of the well-known arbitrariness of the concept of the potential in classical mechanics.

But this arbitrariness of the concept of the potential did not, and does not, exist in quantum mechanics. The electromagnetic vector and scalar potentials were viewed in quantum mechanics in yet another way. Upon the development of quantum mechanics, Weyl and others realized that the original gauge theory could be given a new meaning. They realized that the phase of a wave function could be a new *local* variable. Instead of a change of scale or metric, for which it was originally introduced, a gauge transformation was reinterpreted as a change in the phase of the wave function,

$$\Psi \rightarrow \Psi \exp[-ie^{\lambda}], \tag{2.5}$$

and the gauge transformation for the potential $\mathbf{A}_{\mu}$ became

$$\mathbf{A}_{\mu} \rightarrow \mathbf{A}_{\mu} - \frac{\partial \lambda}{\partial x_{\mu}}. \tag{2.6}$$

Equations (2.5) and (2.6) together ensure that the Schrödinger formulation for a single charged particle in an electromagnetic field remains invariant to phase changes because they self-cancel.[e] Thus any change in location, for that *single* charged particle, which produces a change in the phase [Eq. (2.5)] is compensated for by a corresponding change in the potential [Eq. (2.6)]. Therefore Weyl's original idea, reinterpreted, was accepted, and the potential in quantum mechanics was viewed as a *connection* which relates phases at different locations. Nevertheless, this use and interpretation did not carry

[e]Wignall[255] has shown that no phase change occurs for de Broglie waves under the low velocity limit of the Lorentz transformation. However, the phase of de Broglie waves is not invariant under a change of frame described by a Galilean transformation.

over into classical mechanics and a schizoid attitude has existed to this day regarding the physical meaning of the potentials in classical and quantum mechanics. In classical mechanics the potentials were, up until recently, viewed as having only an arbitrary mathematical, not physical, meaning, as they seemed to lack gauge invariance. In quantum mechanics, however, they *are* viewed as gauge-invariant and *do* possess a physical meaning. It is an aim of this book to show that in classical mechanics the potentials can also be taken to have, under special circumstances, a physical meaning, and possess gauge invariance under certain (topological) circumstances, and gauge covariance under others.

A major impetus to rethink the physical meaning of the potentials in classical mechanics came about from the experiments examined in the next section.

3. Empirical Reasons for Questioning the Completeness of Maxwell's Theory

3.1. *Aharonov–Bohm (AB) and Altshuler–Aronov–Spivak (AAS) effects*

Beginning in 1959 Aharonov and Bohm[40] challenged the view that the classical vector potential produces no observable physical effects by proposing two experiments. The one which is most discussed is shown in Fig. 3.1.1. A beam of monoenergetic electrons exits from the source at X and is diffracted into two beams by the two slits in the wall at Y1 and Y2. The two beams produce an interference pattern at Z which is measured. Behind the wall is a solenoid, S, the **B** field of which points out of the paper. The postulate of the absence of a free local magnetic monopole [Maxwell equation (3) above] predicts the magnetic field outside the solenoid to be zero. Before the current is turned on in the solenoid, there should be the usually expected interference patterns seen at Z. Aharonov and Bohm predicted that if the current is turned on and due to the differently directed **A** fields in paths 1 and 2 indicated by the arrows in Fig. 3.1.1, additional phase shifts should be discernible at Z. This prediction was confirmed experimentally[41–48] and the evidence has been extensively reviewed.[49–53] Aharonov and Casher[54] have extended the theoretical treatment of the AB effect to neutral particles with

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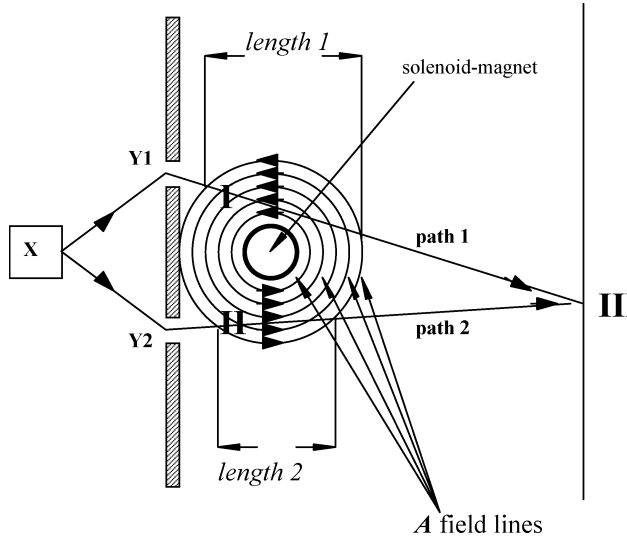

Fig. 3.1.1. Two-slit diffraction experiment of the Aharonov–Bohm effect. Electrons are produced by a source at X, diffracted by the slits at Y1 and Y2, and their diffraction pattern is detected at Z. The solenoid is between the slits and directed out of the page. The different orientations of the **A** field at the points of interaction with the two paths are indicated by the arrows > and < following the right hand rule.

a magnetic moment; and Botelho and de Mello[55] have analyzed a non-Abelian AB effect in the framework of pseudoclassical mechanics.

One explanation of the effect is as follows. Let ψ_0 be the wave function when there is no current in the solenoid. After the current is turned on, the Hamiltonian is

$$H = \frac{1}{2m}(-i\hbar\nabla\psi - e\mathbf{A})^2, \tag{3.1.1}$$

and the new wave function is

$$\psi = \psi_0 \exp\left[\frac{-ieS}{\hbar}\right], \tag{3.1.2}$$

where, S, the flux, is defined as

$$S = \oint \mathbf{A} \cdot d\mathbf{x}, \tag{3.1.3}$$

which is the quantum analog of the classical action evaluated along the paths 1 and 2. At point Z, the wave functions of the two electron

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beams are

$$\psi_1 = \psi_0 \exp\left[\frac{-eS_1}{\hbar}\right], \qquad \psi_2 = \psi_0 \exp\left[\frac{-eS_2}{\hbar}\right], \tag{3.1.4}$$

and the phase difference is

$$\left(\frac{e}{\hbar}\right)(S_1 - S_2) = \left(\frac{e}{\hbar}\right)\left(\int_1 \mathbf{A}\cdot d\mathbf{x} - \int_2 \mathbf{A}\cdot d\mathbf{x}\right) = 2\pi\left(\frac{e}{\hbar}\right)\boldsymbol{\Phi}. \tag{3.1.5}$$

By Stokes' theorem, this is directly proportional to the magnetic flux, $\boldsymbol{\Phi} = \oint \mathbf{A}\cdot d\mathbf{x}$, in the solenoid.

However, the phase difference given by Eq. (3.1.5) is not single-valued. Therefore, the value of the phase change will only be determined to within an arbitrary multiple, n, of $2\pi e\boldsymbol{\Phi}/\hbar$, where n is the number of times the measured charge circulated the solenoid. The topological feature of the background space of the AB effect is its *multiple connectedness*.[56] Therefore, the mathematical object to be computed in this framework is a propagator expressed as a path integral in the covering space of the background physical space (cf. Ref. 57). This means that for a *simply connected* space, all paths between two points are in the same homotopy class, and the effect of the potential, $\mathbf{A}_\mu$, is as a multiplier of the free-particle propagator with a single gauge phase factor. *In this case, the potential has no physically discernible effect*. However, for a *multiply connected* manifold, the potential *can have a physically discernible effect* because the gauge factors can be different for different homotopy classes.[58,59]

The AB effect was confirmed experimentally in the originally proposed field → free electron (F → FE) situation (cf. Ref. 41). That is, the effect predicted by Aharonov and Bohm refers to the influence of the **A** vector potential on electrons confined to a *multiply connected region*, but within which the magnetic field is zero. As a consequence of the gauge invariance, the energy levels of the electrons have a period $\hbar/e$ of the enclosed flux. More recent experiments have addressed the appearance of the effect in the field → conduction electron (F → CE) situation (cf. Ref. 96). This situation is also not strictly the same as in

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the originally proposed AB experiment in another respect — the magnetic flux is produced by a large solenoid surrounding the influenced condensed matter, usually a loop or a cylinder — so that the **B** field is not set to zero within the material. However, the preponderance of the **B** field is always in the hole encompassed by that cylinder or ring, and the magnetic field causes only secondary effects in the material. In this situation, periodic oscillations in the conductance of the ring appear as a function of the applied magnetic field, **B**. The periodicity of the oscillations is

$$\nabla \mathbf{B} = \frac{\hbar}{e\boldsymbol{A}}, \tag{3.1.6}$$

where $\boldsymbol{A}$ is the area enclosed by the ring.

Under these conditions the AB effect is seen in normal metal,[45–47,60–67] bulk Mg,[68] semiconductors,[69,70] and on doubly connected geometries on GaAs/AlGaAs heterostructures.[71] The effect has been seen in structures such as cylindrical Mg films[72,73] and Li films,[74] wire arrays,[75,76] arrays of Ag loops,[77] small metal loops[60,65] and MBE-grown double quantum wells.[66]

Bandyopadhyay *et al.*[78] and Datta and Bandyopadhyay[70] have also discussed a novel concept for a transistor based on the electrostatic AB effect in MBE-grown quantum wells, where the current is modulated by quantum interference of electrons in two contiguous channels of a gate voltage. They predict that transistors based on this effect will have power-delay products that are orders of magnitude better than those of existing devices such as MODFETs and Josephson junctions. The transconductance will also be much higher than that of MODFETs. Unlike previous experimental treatments which assumed diffusive transport with negligible inelastic scattering, Datta and Bandyopadhyay[70] assume ballistic transport and perfect symmetry in the arms of the interferometer and in the voltage along the interferometer or two-channel structure.

Now, the AB (F $\rightarrow$ CE) effect is temperature-dependent as coherent transport is required. The effect has only been seen at very low temperatures. Measurements were made on parallel GaAs quantum wells at 4.2 K and below,[66] on 860 nm-i.d. Au loops at 0.003 K[61] and 0.05 K $< T <$ 0.7 K,[60,63] on 75 nm-o.d. Sb loops at 0.01 $< T <$

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1 K[79] and at 0.04 K,[64] and on Ag loop arrays at 4.2 K.[77] Measurements on 1.5–2.0-micron-diameter Mg cylinders of length 1 cm were made at 1.12 K.[72] The Thouless scaling parameter, V, or the sensitivity of energy levels to a change in the phase of the wave functions at the boundaries[80,81] implies that the necessary energy correlation range for small rings is accessible in the temperature range 0.0001–10 K.[82]

What is remarkable is that these experiments on the (F → CE) AB effect demonstrate that the effect *can* occur in *disordered* electrical conductors if the temperature is low enough. The effect in metals is a small magnetoresistance oscillation superimposed on the ohmic resistance in multiply connected conductors at low temperatures.[72,74,82] *This means that the conducting electrons must possess a high degree of phase coherence (internal correlation) over distances larger than the atomic spacing or the free path length.*[f] It was initially thought that the effects of finite temperature and the scattering from, and collision with, impurities, would cause incoherence and prevent the observation of the AB effect in bulk samples.[83] The metal loops used measure, for example, less than a micron in diameter and less than 0.1 microns in line thickness. Therefore, the electron is thought to be represented by a pair of waves — one traveling around the ring in the clockwise direction, and the other in the opposite direction, but following the time-reversed path of the first wave. Thus, although each wave has been scattered many times, *each wave collides with the same impurities*, i.e. acquires the same phase shifts, resulting in constructive interference at the origin. *The total path length of both waves is twice the circumference of the ring, meeting the requirement that the phase coherence of the electrons be larger than the circumference of the ring*, or, the transport through the metals arms considered as disordered systems is determined by the eigenvalues of a large random matrix.[83]

Thus, the conductance, G, of a one-dimensional ring in the presence of elastic scattering is[84]

$$G = \left(\frac{2e^2}{\hbar}\right)\left(\frac{t}{1-t}\right), \tag{3.1.7}$$

[f]The author is indebted to an anonymous reviewer of this page for pointing out that the same coherence effect exists for electrons circulating in an antenna in the presence of ions. If this coherence effect did not exist, radiation would be impossible since emitted power would be proportional to I instead of I^2.

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and an AB flux applied to the ring results in periodic oscillations of G, *provided that the phase coherence length of the ring is larger than the size of the system.* (The AB oscillations can be suppressed by magnetic fields, vanishing near resistance minima associated with plateaus in the Hall effect.[85]) Dupuis and Montambaux[86] have shown that in the case of the AB effect in metallic rings, the statistics of levels show a transition from the Gaussian orthogonal ensemble (GOE), in which the statistical ensemble shows time reversal, to the Gaussian unitary ensemble (GUE), in which time reversal symmetry is broken.

A related effect is the Altshuler–Aronov–Spivak (AAS) effect.[87] These authors considered an ultrathin normal metal cylindrical shell of moderate length but very small transverse dimensions at low temperature and how the magnetoresistance would depend on the intensity of magnetic flux axially threading the cylinder. They concluded that it would be an oscillating function of the total flux with a period of $\hbar/2e$, i.e. the same as the flux of the superconductive state. The analogous "flux quantum" of the AB effect is $\hbar/e$[60,61] and differs from the AAS situation, which involves coherent "backscattering." The AAS effect has been observed in a 1000-Å-thick magnesium layer on a quartz fiber several millimeters long.[72] More recent treatments of the AAS effect[82,88,89] are based on the quantum-mechanical transmission (t) coefficients of electrons and, unlike the original AAS treatment, find an $\hbar/e$ periodic component as well as the $\hbar/2e$ harmonic. Raising the temperature above a crossover, T_c, changes the flux periodicity of magnetic resistance oscillations from $\hbar/e$ to $\hbar/2e$, where T_c is determined by the energy correlation range $\hbar D/L^2$, where D is the elastic diffusion constant, L is the length of the sample and the quantity $\hbar D/L^2$ is the Thouless scaling parameter V for a metal.

The AAS effect arises because of a special set of trajectories — time-reversed pairs which form a closed loop — which have a fixed relative phase for any material impurity configuration. These trajectories do not average to zero and contribute to the reflection coefficients which oscillate with period $\hbar/2e$. The $\hbar/e$ oscillations of the AB effect, on the other hand, arise from oscillations in the transmission coefficients and can at higher temperatures, average to zero. Below T_c, both contributions are of order $e^2/\hbar$.[82]

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Xie and DasSarma[90] studied the AB and AAS effects in the transport regime of a strongly disordered system in which electron transport is via a hopping process — specifically via variable-range-hopping transport. Their numerical results indicate that only the $\hbar/2e$ (AAS) flux-periodic oscillations survive at finite temperatures in the presence of any finite disorder.

The results of the metal loop experiments demonstrated that elastic scattering does not destroy the phase memory of the electron wave functions.[61,91] The flux periodicity in a condensed matter system due to the AB effect would not be surprising in a superconductor. However, the same periodicities in *finite* conductors is remarkable.[90] Numerical simulation of variable-range-hopping conduction[90] only finds AB oscillations ($\phi_0 = \hbar/e$) in hopping conductance when a metal ring is small and at low temperature. At the large ring limit and higher temperature AAS ($\phi_0 = \hbar/2e$) oscillations survive — a finding consistent with the experimental findings of Polyarkov *et al.*[92] A suggested reason for the retention of long range phase coherence is that the phase memory is only destroyed exponentially as $\exp[-L/L_i]$, where L_i is a "typical inelastic scattering length" and the destruction depends on the energy changes in the hopping process dependent on long wavelength, low energy acoustic phonons. The search for an explanation for both AB and AAS effects has resulted in consideration of systems as neither precisely quantum-mechanical nor classical, but in between, i.e. "mesoscopic."

"Mesoscopic" systems have been studied by Stone[93] in which the energy and spacing is only a few orders of magnitude smaller than kT at low temperatures. The prediction was made[93] that large AB oscillations should be seen in the transport coefficients of such systems. Such systems have a sample length which is much longer than the elastic mean free path, but shorter than the localization length. The magnetic field through a loop connected to leads changes the relative phase of the contribution from each arm of the loop by $2\pi\Phi/\Phi_0$, where $\Phi_0 = \hbar c/e$ is the one electron flux quantum and Φ is the flux through the hole in the loop — but only if the phase-dependent terms do not average to zero. In the mesoscopic range, if inelastic scattering is absent, these phase-dependent contributions do not self-average to zero.

Washburn *et al.*[63] and Stone and Imry[82] demonstrated experimentally that the amplitude of aperiodic and periodic conductance fluctuations decrease for the F → CE AB effect with increasing temperature. There is a characteristic correlation energy:

$$E_C = \frac{\pi\hbar D}{2L^2}, \tag{3.1.8}$$

where D is the diffusion constant of the electrons, and L is the minimum length of the sample. If thermal energy $k_B T > E_C$, the conductance fluctuations decrease as $(E_C/k_B T)^{1/2}$. The conductance fluctuations also decrease when L_ϕ, the phase coherence length, is shorter than the length, L, or the distance between voltage probes, the decrease being described by a factor $\exp(-L/L_\phi)$.[79] This gives a conductance fluctuation:

$$\Delta Gn = \left(\frac{L_\phi}{L}\right)^{3/2}. \tag{3.1.9}$$

In condensed matter, therefore, the AB effect appears as the modulation of the electron wave functions by the $\mathbf{A}_\mu$ potential. The phase of the wave function can also be changed by the application of an electric field.[64] The electric field contributes to the fourth term in the four-vector product $\mathbf{A}_\mu(dx)^\mu$, which contains the scalar potential Φ associated with transverse electric fields and time. The phase shift in the wave function is

$$\Delta\phi = \int \frac{e\Phi dt}{\hbar}. \tag{3.1.10}$$

Experiments on Sb metal loop devices on silicon substrate have demonstrated that the voltage on capacitative probes can be used to tune the position (phase) of $\hbar/e$ oscillations in the loop. Thus, there appear to be two ways to modulate the phase of electrons in condensed matter: application of the $\mathbf{A}_\mu$ potential by threading magnetic flux between two paths of electrons; and application of a scalar potential by means of a transverse electric field. AB fluctuations in metal loops are also not symmetric about $H = 0$: four-probe measurements yield resistances which depend on the lead configurations.[62]

More recently, it has been shown that, owing to transport via edge states and penetration of a strong magnetic field into the conducting

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region, periodic magnetoconductance oscillations can occur in a singly connected geometry, for example as in a point contact or a "quantum dot" (a disk-shaped region in a two-dimensional electron gas).[94,95] As this effect is dependent on transport via edge states circulating along the boundary of a quantum dot and enclosing a well-defined amount of flux, the geometry is effectively doubly connected. The claim of singly-connectedness is thus more apparent than real. However, there is a difference between the AB effect in a ring and in a dot: in each period $\nabla\mathbf{B}$, the number of states below a given energy, stays constant in a ring, but increases by one in a quantum dot. In the case of a dot, the AB magnetoconductance oscillations are accompanied by an increase in the charge of the dot by one elementary charge per period. This can result in an increase in Coulomb repulsion which can block the AB magnetoconductance oscillations. This effect, occurring in quantum dots, has been called the Coulomb blockade of the AB effect.[94,95]

Finally, Boulware and Deser[97] explain the AB effect in terms of a vector potential coupling minimally to matter, i.e. a vector potential not considered as a gauge field. They provide an experimental bound on the range of such a potential as 10^2 km.

In summary, the AB and AAS effects, whether F $\rightarrow$ FE or F $\rightarrow$ CE, demonstrate that the phase of a *composite* particle's wave function is a physical degree of freedom which is dependent on differences in $\mathbf{A}_\mu$ potential influences on the space–time position or path of a *first* particle's wave function with respect to that of another, *second*, particle's wave function. But the *connection*, or *mapping*, between spatiotemporally different fields or particles which originated at, or passed through, spatiotemporally separated points or paths with differential $\mathbf{A}_\mu$ potential influences, is only measurable by many-to-one mapping of those different fields or particles. By interpreting the phase of a wave function as a *local* variable instead of the norm of a vector, electromagnetism can be interpreted as a *local* gauge (phase) theory, if not exactly, then very close to the way which Weyl originally envisioned it to be.

Below, the interaction of the $\mathbf{A}_\mu$ field (x), whether vector or potential, as an independent variable with dependent variable constructs will be referred to in an $x \rightarrow y$ notation. For example, field $\rightarrow$ free electron,

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field → conducting electron, field → wave guide, field → neutral particle and field → rotating frame interactions will be referred to as (F → FE), (F → CE), (F → WG), (F → P) and (F → RF) interactions. Although the $\mathbf{A}_\mu$ field is a classical field, the AB and AAS effects are either F → FE or F → CE effects and might be considered "special" in that they involve quantum-mechanical particles, i.e. electrons. In the next section, however, we examine a phase rotation which can only be considered classical, as *both* independent and dependent variables are classical. Nonetheless, the same result — the $\mathbf{A}_\mu$ potentials demonstrate physical effects — applies.

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3.2. *Topological phases: Berry, Aharonov–Anandan, Pancharatnam and Chiao–Wu phase rotation effects*

When addressing the AB effect, Wu and Yang[98] argued that the wave function of a system will be multiplied by a nonintegrable (path-dependent) phase factor after its transport around a closed curve in the presence of an $\mathbf{A}_\mu$ potential *in ordinary space*. In the case of the Berry–Aharonov–Anandan–Pancharatnam (BAAP) phase, another nonintegrable phase factor arises from the adiabatic transport of a system around a closed path in *parameter (momentum) space*, i.e. this topological phase is the AB effect in parameter space.[99–108] The WG → F version of this effect has been experimentally verified[109] and the phase effect in general interpreted as being due to parallel transport in the presence of a gauge field.[110] The effect exists at both the classical and the quantum level (cf. Refs. 111 and 112).

There has been, however, an evolution of understanding concerning the origins of topological phase effects. Berry[99] originally proposed a geometrical (beside the usual dynamical) phase acquisition for a nondegenerate quantum state which varies adiabatically through a circuit in parameter space. Later, the constraint of an adiabatic approximation was removed[103] and also the constraint of degenerate states.[28] Then Aharonov and Anandan[113] showed that the effect can be defined for any cyclic evolution of a quantum system. Bhandari and Samuel[114] have also pointed out that Berry's phase is closely connected with a phase discovered by Pancharatnam.[115] These authors

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also demonstrated that unitary time evolution is not essential for the appearance of a phase change in one beam of an interferometer. This is because the polarization state of light can be taken along a closed circuit on the Poincaré sphere, and the resulting polarization changes are not necessarily a function of a unitary time evolution. Thus, the current thinking is that the history of "windings" of a particle is "remembered," or registered and indicated, by changes in phase either in a quantum-mechanical particle's state or in a classical wave's polarization. The topological phase effect appears to arise from the nontrivial topology of the complex projective Hilbert space — whether classical or quantum-mechanical[116] — and to be equivalent to a gauge potential in the parameter space of the system — again, whether classical or quantum-mechanical.

Jiao *et al.*[117] have also indicated at least three variations of topological phases: (i) the phase which arises from a cycling in the direction of a beam of light so that the tip of the spin vector of a photon in this beam traces out a closed curve on the sphere of spin directions — which is that originally studied by Chiao and Wu[106]; (ii) Pancharatnam's phase arising from a cycling in the polarization states of the light while keeping the direction of the beam of light fixed, so that Stokes' vector traces out a closed curve on the Poincaré sphere, i.e. the phase change is due to a polarization change; (iii) the phase change due to a cycle of changes in squeezed states of light.

Topological phase change effects, in the field $\rightarrow$ photon version, have been observed in NMR interferometry experiments[118,119] and using ultracold neutrons[120]; in coherent states[121,122]; optical resonance[123]; and the degenerate parametric amplifier.[124] However, topological phase change effects are more commonly studied in a classical wave guide $\rightarrow$ classical field (WG $\rightarrow$ F) version, in which the parameter space is the momentum $\mathbf{k}$ space.[106,117,125]

For example, the helicity or polarization state, σ, is[106]

$$\sigma = \mathbf{s} \cdot \mathbf{k}, \tag{3.2.1}$$

where $\mathbf{s}$ is a spin or helicity operator and $\mathbf{k}$ is the direction of propagation (k_x, k_y, k_z). If τ is the optical path length, then $|\mathbf{k}(\tau), \sigma >$ is the spin or polarization state. Interpreted classically, the constraint of keeping $\mathbf{k}$ parallel to the axis of a wave guide is due to the linear momentum

being in that direction. This means that a wave guide can act as a polarization rotator. Furthermore, as helicity (polarization), σ, is adiabatically conserved, **s** is also constrained to remain parallel to the local axis of the wave guide. Therefore, the topology of a wave guide, e.g. a helix shape, will constrain **k**, and also **s**, to perform a trajectory C on the surface of a sphere in the parameter space (k_x, k_y, k_z) which prescribes the linear momentum. Thus, the topology of the constrained trajectory of radiation progressing between two *local* positions has a *global* effect indicated by a polarization (spin) change. If $\gamma(C)$ is the topological phase, and $\beta = \exp[i\gamma(C)]$ is a phase factor, the final polarization state after progression along a constrained trajectory, i.e. "momentum conditioning," is

$$\sigma_2 = \beta \cdot \mathbf{s} \cdot \mathbf{k} \tag{3.2.2}$$

where the subscript indicates a second location on the trajectory.

As a monopole is theoretically required at $\mathbf{k} = 0$, owing to the radial symmetry of the parameter space and resulting singularity, a solid angle $\Omega(C)$ can be defined on a parameter space sphere with respect to the origin $\mathbf{k} = 0$. Thus, $\Omega(C)$ can be said to define the "excited states" of the monopole at $\mathbf{k} = 0$. Therefore

$$\sigma_2 - \sigma_1 = \beta \cdot \mathbf{s} \cdot \mathbf{k} - \sigma_1 = \sigma_1 \Omega(C) - \sigma_1 = \gamma(C). \tag{3.2.3}$$

The following question can be asked: What conservation law underlies the topological phase? A clue is provided by Kitano *et al.*,[126] who point out that the phase change can also be seen in discrete optical systems which contain no wave guides, for example in a configuration of (ideal or infinitely conducting) mirrors. Now, mirrors do not conserve helicity; they reverse it and the local tangent vector, **t**, must be replaced by $-\mathbf{t}$ on alternate segments of the light path. Mirror configurations of this type have been used in a laser gyro.[127] This suggests that changes of acceleration, whether along a wave guide or in a mirror reflection, under equivalence principle conditions are the compensatory changes which match changes in the topological phase, giving the conservation equation:

$$\gamma(C) + \oint \mathbf{A} \cdot dl = 0. \tag{3.2.4}$$

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That the phase effect change can occur in classical mechanical form is witnessed by changes in polarization rotation resulting from changes in the topological path of a light beam. Tomita and Chiao[109] demonstrated effective optical activity of a helically wound single mode optical fiber in confirmation of Berry's prediction. The angle of rotation of linearly polarized light in the fiber gives a direct measure of the topological phase *at the classical level*. (Hannay[128] has also discussed the classical limit of the topological phase in the case of a symmetric top.) The effect arises from the overall geometry of the path taken by the light and is thus a *global topological effect* independent of the material properties of the fiber. The optical rotation is *independent of geometry* and therefore may be said to quantify the "*topological charge*" of the system, i.e. the helicity of the photon, which is a relativistic quantum number.

Referring to Fig. 3.2.1, the fiber length is

$$s = [p^2 + (2\pi r)^2]^{1/2}, \tag{3.2.5}$$

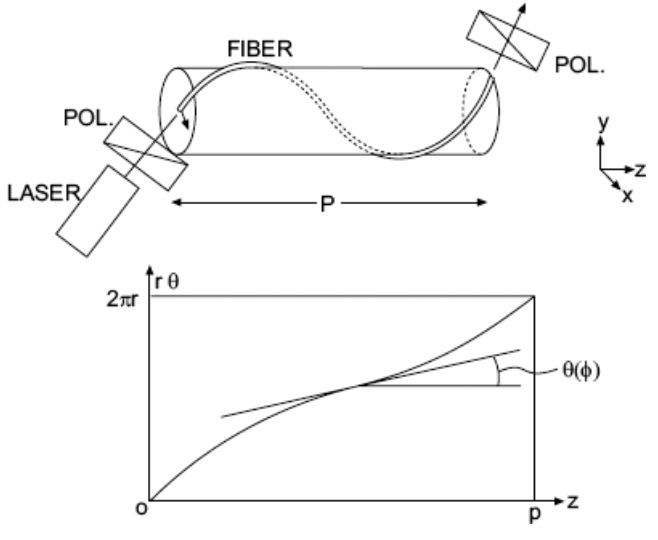

Fig. 3.2.1. (a) Experimental setup; (b) geometry used to calculate the solid angle in momentum space of a nonuniformaly wound fiber on a cylinder. (After Ref. 109.)

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and the solid angle in momentum space $\Omega(C)$ spanned by the fiber's closed path C, a circle in the case considered, is

$$\Omega(C) - 2\pi(1 - \cos\theta). \tag{3.2.6}$$

The topological phase is

$$\gamma(C) = -2\pi\sigma\left(1 - \frac{p}{s}\right), \tag{3.2.7}$$

where $\sigma = \pm 1$ is the helicity quantum number of the photon.

By wrapping a piece of paper with a computer-generated curve on a cylinder to which the fiber is fitted, and then unwrapping the paper, the local pitch angle, or the tangent to the curve followed by the fiber, can be estimated to be [Fig. 3.2.1(b)]

$$\theta(\phi) = \tan^{-1}\left(\frac{rd\phi}{dz}\right), \tag{3.2.8}$$

which is the angle between the local wave guide and the helix axes. In momentum space, $\theta(\phi + \pi/2)$ traces out a closed curve C, the fiber path on the surface of a sphere. The solid angle subtended by C to the center of the sphere is

$$\Omega(C) = \int_0^{2\pi} [1 - \cos\theta(\phi)]d\phi. \tag{3.2.9}$$

The topological phase is then, more correctly,

$$\gamma(C) = -\sigma\,\Omega(C) \quad \text{or}$$
$$\gamma(C) = \nu\,\Omega(C) \tag{3.2.10}$$

(where $\nu = 1/2$ in the case of polarization charges, i.e. Pancharatnam's phase). There is thus a linear relation between the angle of rotation of linearly polarized light, and the solid angle $\Omega(C)$ subtended by C at the origin of the momentum space of the photon.[109]

More recently, Chiao *et al.*[129] have demonstrated a topological phase shift in a Mach–Zehnder interferometer in which light travels along nonplanar paths in two arms. They interpret their results in terms of the Aharonov–Anandan phase and changes in projective Hilbert space, i.e. the sphere of spin directions of the photon, rather

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than parameter (momentum) space. The hypothesis tested was that the evolution of the state of a system is cyclic, i.e. that it returns to its starting point adiabatically or not. Thus the C in Eq. (3.2.10) is to be interpreted as a closed circuit on the sphere of spin directions.

Chiao and Wu[106] consider topological phase rotation effects to be "topological features of the Maxwell theory which originate at the quantum level, but survive the correspondence principle limit ($h \to 0$) into the classical level." However, this opinion is contested and the effect is viewed as classical by other authors (e.g. Refs. 102, 103, 130 and 131). For example, the evolution of the polarization vector can be viewed as being determined by a connection on the tangent bundle of the two-dimensional sphere.[130,131] The effect is then viewed as non-Abelian. The situation can then be described with a family of Hamiltonian operators, $H_0 + \mathbf{k} \cdot \mathbf{V}$, where H_0 is rotationally invariant, $\mathbf{V}$ is a vector operator and $\mathbf{k}$ varies over the unit vectors in $\mathbf{R^3}$.[131]

The Chiao–Wu phase and the Pancharatnam phase are additive.[117] This is because the two topological phase effects arise in different parameter spaces: the former in $\mathbf{k}$ space and the latter in polarization vector (Poincaré sphere) space. To see this, Maxwell's equations can be recast into a six-component spinor form[117]

$$\nabla \times (\mathbf{E} \pm \mathbf{B}) = \frac{\pm \partial(\mathbf{E} \pm i\mathbf{B})}{\partial t}, \tag{3.2.11}$$

$$\Psi = \mathbf{col}(\mathbf{E} + \mathbf{B}, \mathbf{E} - \mathbf{B}) = \mathbf{col}((\Psi^+), (\Psi^-)), \tag{3.2.12}$$

where **col** denotes a column vector, to obtain a Schrödinger-like equation

$$\frac{i\partial\Psi}{\partial t} = H\Psi, \tag{3.2.13}$$

where the Hamiltonian, H, is given by

$$H = \begin{matrix} (\nabla x) & 0 \\ 0 & (-\nabla x) \end{matrix} \tag{3.2.14}$$

(∇x represents the curl). This spinor representation of Maxwell's equations has a natural correspondence with natural optical activity in the frequency domain.[132]

The conventional dynamical phase becomes

$$\delta(H) = -\int_0^T (\Psi, H\Psi)dt \tag{3.2.15}$$

and the geometrical (topological) phase is, as before

$$\gamma(C) = \oint \mathbf{A} \cdot dl, \tag{3.2.16}$$

but where the vector potential, **A**, is explicitly defined as

$$A = -(\Psi, \nabla\Psi), \tag{3.2.17}$$

i.e. a connection defined on the state space.

By means of Stokes' theorem

$$\gamma(C) = \oint \mathbf{A} \cdot dl = \int_S \nabla \times AdS, \tag{3.2.18}$$

$$\gamma(C) = \nu\,\Omega(C), \tag{3.2.19}$$

as before.

Figure 3.2.2 shows the different manifestations and representations of the topological phase effect. Figure (i) is a sphere of *spin directions* for representing the (Chiao–Wu) phase arising from the spin vector of a photon tracing out a closed curve on the sphere.[117] The topological phase is equal to the angle Ω. Figure (ii) is a Poincaré sphere of polarization states, or *helicity*, of a photon for representing the (Pancharatnam) phase effect arising from cycling in the polarization states of the photon while keeping the direction of the beam fixed.[114] The topological phase is equal to the *negative* of one half the angle Ω. Figure (iii) is a *generalized* Poincaré sphere for representing the angular momentum of light (with space-fixed axis),[117] or a *null flag or twistor* representation[34,133] for representing the Pancharatnam topological phase but with the phase equal to the *positive* value of one half the angle Ω. The topological phase effects represented in (i) and (iii) are additive.

A more explicit relation between the topological phase effect and Maxwell's theory is obtained within the formulation of Maxwell's

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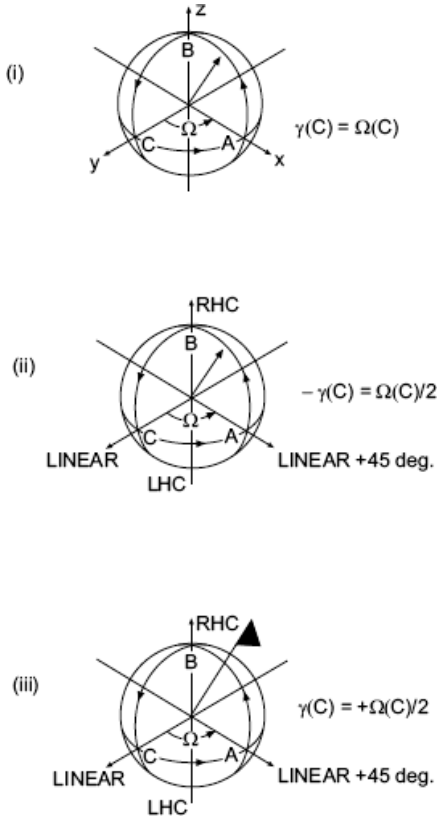

Fig. 3.2.2. (i) A sphere of *spin directions* for representing the (Chiao–Wu) phase arising from the spin vector of a photon tracing out a closed curve on the sphere. (After Ref. 117.) The topological phase is equal to the angle Ω. (ii) A Poincaré sphere of polarization states, or *helicity*, of a photon for representing the (Pancharatnam) phase effect arising from cycling in the polarization states of the photon while keeping the direction of the beam fixed (After Ref. 114). The topological phase is equal to the *negative* of one half the angle Ω. (iii) A *generalized* Poincaré sphere for representing the angular momentum of light (with space-fixed axis) (after Ref. 117), or a *null flag or twistor* representation (after Refs. 34 and 133) for representing the Pancharatnam topological phase but with the phase equal to the *positive* value of one half the angle Ω. The topological phase effects represented in (i) and (iii) are additive.

theory by Biakynicki-Birula and Bialynicka-Birula.[134] Within this formulation, the intrinsic properties of an electromagnetic wave are its wave vector, **k**, and its polarization, **e**(**k**). As Maxwell's theory can be formulated as a representation of Poincaré symmetry, all wave vectors

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form a vector space. Thus, implicit in Maxwell's theory is the topology of the surface of a sphere (i.e. the submanifold, S^2). The generators of the Poincaré group involve a covariant derivative in momentum space, whose curvature is given by a magnetic monopole field. However, the Maxwell equations can only determine the polarization tensor, $\mathbf{e}(\mathbf{k})$, *up to an arbitrary phase factor*, as Maxwell's theory corresponds to the structure group U(1), i.e. Maxwell's theory *cannot* determine the phase of polarization in momentum space. This arbitrariness permits additional topological phase effects.

On the other hand, the topological phase is *precisely* obtained from a set of angles associated with a group element and there is just one such angle corresponding to a holonomy transformation of a vector bundle around a closed curve on a sphere.[135] The parameter space is the based manifold and each fiber is isomorphic to an N-dimensional Hilbert space. In particular, for the SU(2) case there is a single angle from the holonomy of the Riemannian connection on a sphere.[135] The observation that gauge structure appears in simple dynamical systems — both quantum-mechanical and classical — has been made.[28] For the special case of Fermi systems, the differential geometric background for the occurrence of SU(2) topological phases is the quaternionic projective space with a time evolution corresponding to the SU(2) Yang–Mills instanton.[136] *Locally*, the non-Abelian phase generated can be reduced to an Abelian form. However, it is not possible to define the connection defined on the bundle space except *globally*. This reflects the truly non-Abelian nature of the topological phase. The topological phase effect can be described in a *generalized* Bloch sphere model and an SU(2) Lie group formulation in the spin-coherent state.[137] Furthermore, while acknowledging that in general terms, and formally, Berry's phase is a geometrical object in projective Hilbert space (ray space), the nonadiabatic Berry's phase, physically, is related to the expectation value of spin (spin alignment), and Berry's phase quantization is related to spin alignment quantization.[138]

The topological phase effect even appears in quantum systems constrained by molecular geometry. For example, the topological phase effect appears in the molecular system Na_3.[139] Suppose that a system in an eigenstate $C(r, t)$ responds to slowly varying changes in its

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parameters $R(t)$, such that the system remains in the *same eigenstate apart from an acquired phase*. If the parameters, $R(t)$, completed a circuit in parameter space, then that acquired phase is not simply the familiar dynamical phase, $[(i\hbar)^{-1}E(R(t)]\,dt$, but an additional geometrical phase factor, $\gamma_n(c)$. The origins of this additional phase factor depend only on the geometry of the parameter space and the topology of the circuit traversed. Therefore, adiabatic excursions of molecular wave functions in the neighborhood of an electronic degeneracy result in a change of phase. That is, if the internuclear coordinates of a wave function traverse a circuit in which the state is degenerate with another, then the electronic wave function acquires an additional phase, i.e. it changes its sign. This change was predicted[140–142] and is a special case of the topological phase applying to a large class of molecular systems exhibiting conical intersections. Delacrétaz *et al.*[139] reported the evidence for half-odd quantization of free molecular pseudorotation and offered the first experimental confirmation of the sign change theorem and a direct measurement of the phase. The topological phase has also been observed in fast-rotating superfluid nuclei, i.e. oscillations of pair transfer matrix elements as a function of the angular velocity[143] and in neutron spin rotation.[144]

The appearance of the topological phase effect in both classical and quantum-mechanical systems thus gives credence to the view that the $\mathbf{A}_\mu$ potentials register physical effects at both the classical and the quantum-mechanical level. That such a role for these potentials exists at the quantum-mechanical level is not new. *It is new to consider the* $\mathbf{A}_\mu$ *potentials for such a role at the classical level*. One may ask how the schism in viewing the $\mathbf{A}_\mu$ potentials came about, i.e. why are they viewed as physical constructs in quantum mechanics, but as merely arbitrary mathematical conveniences in classical mechanics? *The answer is that whereas quantum theory is defined with respect to boundary conditions, in the formal presentation of Maxwell's theory boundary conditions are undefined*. Stokes' theorem demonstrates this.

3.3. *Stokes' theorem re-examined*

Stokes' theorem of potential theory applied to classical electromagnetism relates diverging potentials on line elements to rotating

potentials on surface elements. Thus, Stokes' theorem describes a local-to-global field relationship.

If $\mathbf{A}(x)$ is a vector field, S is an open, orientable surface, C is the closed curve bounding S, $\mathbf{dl}$ is a line element of C, $\mathbf{n}$ is the normal to S and C is traversed in a right-hand screw sense (positive direction) relative to $\mathbf{n}$, then the line integral of A is equal to the surface integral over S of $(\nabla \times \mathbf{A}) \cdot \mathbf{n}$:

$$\oint \mathbf{A} \cdot \mathbf{dl} = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n}\, da. \tag{3.3.1}$$

It is also necessary that S be the union of a finite number of smooth surface elements and that the first order partial derivation of the components of $\mathbf{A}$ be continuous on S. Thus Stokes' theorem, as described, takes no account of: (i) space–time overlap in a region with fields derived from different sources; (ii) the exact form of the boundary conditions.

This neglect of the exact form of the boundary conditions in Stokes' theorem of classical mechanics can be contrasted with the situation in quantum mechanics. In quantum mechanics, the wave function satisfies a partial differential equation coupled to boundary conditions because the Schrödinger equation describes a minimum path solution to a trajectory between two points. The boundary condition in the doubly connected (overlap) region outside of the shielded volume in an AB experiment is the reason for the single-valuedness of the wave function, and also the reason for quantization. The situation is also different with spatial symmetries other than the usual, Abelian, spatial symmetry.

A non-Abelian Stokes theorem is[145]

$$h^{-1}\left(\frac{dh}{ds}\right) \approx ie \int_0^1 g^{-1}\mathbf{G}_{ij} g \left(\frac{\partial r^i}{\partial t}\right)\left(\frac{\partial r^j}{\partial s}\right) dt, \tag{3.3.2}$$

where $h(s)$ is a path-dependent phase factor associated with a closed loop and defines a closed loop $r(s, t)$, $0 \leq t \leq 1$, s fixed, in the U(1) symmetry space, $\mathbf{H}$ (equivalent to $\mathbf{A}_\mu$); $\mathbf{G}$ is a gauge field tensor for the SU(2) non-Abelian group; and g is magnetic charge. Here, the boundary conditions, i.e. the path dependencies, are made explicit,

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and we have a local field [with U(1) symmetry] to global field [with SU(2) symmetry] connection.

In classical electromagnetism, therefore, Stokes' theorem appears merely as a useful mathematical relation between a vector field and its curl. *In gauge theory, on the other hand, an amended Stokes' theorem would provide the value for the net comparative phase change in the internal direction of a particle traversing a closed path, i.e. a local-to-global connection.*

Lest it be thought that the $\mathbf{A}_\mu$ field which functions as the independent variable in the AB experiment is only a quantum effect with no relevance to classical behavior, the relation of the $\mathbf{A}_\mu$ potential to the properties of bulk condensed matter is examined in the following section. A more complete definition of Stokes' theorem is also given in Sec. 4 below [Eq. (4.10)].

Use of Stokes' theorem has a price: that of the (covert) adoption of a gauge for local-to-global connections. This is because Stokes' theorem applies directly to propagation issues, which are defined by local-to-global connections. Such connections are also required in propagation through matter. Thus, there is a requirement for Stokes' theorem in any realistic definition of macroscopic properties of matter, and in the next section we see that the physical effects of the $\mathbf{A}_\mu$ potentials exist not merely in fields traversing through various connecting topologies, but in radiation–matter interactions.

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3.4. *Properties of bulk condensed matter — Ehrenberg and Siday's observation*

In the AB F $\rightarrow$ FE situation, when the size of the solenoid is much larger than the de Broglie wavelength of the incident electrons, the scattering amplitude is essentially dominated by simple classical trajectories. But the classical manifestation of quantum influences is not peculiar to the AB effect. For example, macroscopic quantum tunneling is observable in Josephson tunnel junctions in which the phase difference of the junction can be regarded as a macroscopic degree of freedom, i.e. a classical variable.[146,147]

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Even without known quantum influences or quantum-mechanical explanation, there is a classical justification for the $\mathbf{A}_\mu$ potential as a physical effect. For example, on the basis of optical arguments, the $\mathbf{A}_\mu$ potential must be chosen so as to satisfy Stokes' theorem, *thereby removing the arbitrariness with respect to the gauge*. Furthermore, an argument originating with Ehrenberg and Siday[148] shows that a gauge-invariant $\mathbf{A}_\mu$ potential is *presupposed* in any definition of the refractive index.

This argument is a derivation of the refractive index based on *Fermat's principle*: in any optical medium, a scalar quantity, e.g. the refractive index, finite everywhere in space, can be defined so that the line integral in the three-dimensional space taken between any fixed points must be an extremum which passes through these points. The optical path along a given line connecting a point 1 and a point 2 is

$$\int_1^2 m ds = \int_1^2 [mv + (\mathbf{A}_\mu \cdot \mathbf{n})]\, ds, \tag{3.4.1}$$

where $\mathbf{n}$ is the unit vector in the direction of the line, v is the velocity of the electron, and m is its mass. Defined in this way, an unambiguous definition of the refractive index indicates the *necessity* of a unique (gauge-invariant) definition of the $\mathbf{A}_\mu$ potential. Stated differently: an unambiguous definition of the refractive index implies defining the boundary conditions through which test radiation moves. These boundary conditions define a definite gauge and thereby definite $\mathbf{A}_\mu$ potentials.

This an example of physical $\mathbf{A}_\mu$ -dependent effects (the refractive properties of matter) seen when radiation propagates through matter — from one point to another. In the next section $\mathbf{A}_\mu$ effects are described when two fields are in close proximity. This is the Josephson effect, and again, the potential functions as a global-to-local operator.

3.5. *The Josephson effect*

Josephson[149–152] predicted that a d.c. voltage, V, across the partitioning barrier of a superconductor gives rise to an alternating current of

frequency

$$\omega = \frac{2eV}{\hbar}. \tag{3.5.1}$$

The equivalent induced voltage is[153]

$$V = \left(\frac{1}{c}\right)\frac{d\mathbf{\Phi}}{dt}, \tag{3.5.2}$$

where $\mathbf{\Phi}$ is the magnetic flux through a superconducting ring containing a barrier. The circulating current, I, exhibits a periodic dependence upon $\mathbf{\Phi}$

$$I(\alpha) = \sum_n a_n \sin 2\pi\alpha, \tag{3.5.3}$$

where

$$\alpha = \frac{\mathbf{\Phi}}{\frac{\hbar c}{e}}. \tag{3.5.4}$$

The validity of Eq. (3.5.4) depends upon the substitution of

$$\mathbf{p} - \frac{e\mathbf{A}}{c} \tag{3.5.5}$$

for the momentum, **p**, of any particle with charge and with a required gauge invariance for the **A** potential.

The phase factor existing in the junction gap of a Josephson junction is an exponential of the integral of the **A** potential. The fluxon, or the decrementlessly conducting wave in the long Josephson junction and in a SQUID, is the equivalent of an **A** wave in one-dimensional phase space. The phenomenological equations are

$$\frac{\partial\phi}{\partial x} = \left(\frac{2ed}{\hbar c}\right)H_y, \tag{3.5.6}$$

$$\frac{\partial\phi}{\partial t} = \left(\frac{2e}{\hbar}\right)V, \tag{3.5.7}$$

$$J_x = j\sin\phi + \sigma V, \tag{3.5.8}$$

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where ϕ is the phase difference between two superconductors; H is the magnetic field in the barrier; V is the voltage across the barrier; $d = 2\lambda + l$; λ is the penetration depth; and l is the barrier thickness.[g]

If the barrier is regarded as having a capacitance, $\mathbf{C}$, per unit area, then Eq. (3.5.6) and Maxwell's equations give

$$\left[\frac{\partial^2}{\partial x^2} - \left(\frac{1}{\mathbf{c}^2}\right)\left(\frac{\partial^2}{\partial t^2}\right) - \left(\frac{\beta}{\mathbf{c}^2}\right)\left(\frac{\partial}{\partial t}\right)\right]\phi = \left(\frac{1}{\lambda_0^2}\right)\sin\phi, \qquad (3.5.9)$$

where $\mathbf{c}^2 = c^2/4\pi dC$ is the phase velocity in the barrier, $\lambda_0^2 = hc^2/\delta\pi edj$ is the penetration depth and $\beta = 4\pi d\mathbf{c}^2\sigma = \sigma/C$ is the damping constant. Anderson[250] demonstrated that solutions to this equation, representing vortex lines in the barrier, are obtained as solutions to

$$\frac{\partial\phi^2}{\partial x^2} = \left(\frac{1}{\lambda_0^2}\right)\sin\phi, \qquad (3.5.10)$$

which, except for the sign, is the equation of a pendulum.

The Josephson effect is remarkable in the present context for three reasons: (i) with well-defined boundary conditions (the barrier), the phase, ϕ, is a well-defined gauge-invariant variable; (ii) an equation of motion can be defined in terms of the well-studied pendulum,[154] relating a phase variable to potential energy; (iii) the "free" energy in the barrier is[155]

$$F = \left(\frac{\hbar j}{2e}\right)\int dx\left[(1-\cos\phi) + \frac{1}{2\lambda_0^2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\lambda_0}{\mathbf{c}}\right)^2\left(\frac{\partial\phi}{\partial t}\right)^2\right], \qquad (3.5.11)$$

an equation which provides an (free) energy measure in terms of the differential of a phase variable. The Josephson effect, like the AB effect, demonstrates the registration of physical influences by means of phase changes. The Josephson phase, also like the AB phase, registers field influences.

[g]Lenstra *et al.*[254] have shown an analogy between Josephson-like oscillations and the Sagnac effect.

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Jaklevic *et al.*[156] studied multiply connected superconductors utilizing Josephson junction tunneling and modulated the supercurrent with an applied magnetic field. The interference "fringes" obtained were found to occur even when the magnetic flux is confined to a region not accessible to the superconductor, i.e. there occurs vector potential modulation of superconducting electron drift velocity. As always, the superconductive state had global phase coherence, indicating that the modulation effect studied was a local ($\mathbf{A}_\mu$) influence on global phase effects (i.e. the phase order parameter in the barrier).

In the case of the next effect examined, the quantized Hall effect, the effect is crucially dependent upon the gauge invariance of the $\mathbf{A}_\mu$ potential. The result of such gauge invariance is remarkably significant: *an independence of the quantization condition on the density of mobile electrons in a test sample.*

This independence was seen above while examining the remarkable independence in preservation of phase coherence in electrons over distances larger than the atomic spacing or the free path length in the F $\rightarrow$ CE AB effect. In both cases, the primacy of macroscopic and "mesoscopic," effects are indicated.

3.6. *The quantized Hall effect*

The quantized Hall effect[157,158] has the following attributes:

(1) There is the presence of a Hall conductance σ_{xx} in a two- dimensional gas within a narrow potential well at a semiconductor — heterostructure interface e.g. in MOS, quantum well and MOSFET;
(2) The temperature is low enough that the electrons are all in the ground state of the potential well and with the Fermi level being between the Landau levels;
(3) The conductance is quantized with a plateau having $\sigma_{xy} = n\hbar/e^2\sigma_{xy}$ (n is an integer) for finite ranges of the gate voltage in which the regular conductance is severely reduced;
(4) Together with the well-known Hall effect (1879) condition (a magnetic field perpendicular to the plane and an electric field in the plane and the electrons drifting in the direction $\mathbf{E} \times \mathbf{B}$), the

energy associated with the cyclotron motion of each electron takes on quantized values $(n + 1/2)\hbar\omega_c$, where ω_c is the cyclotron frequency at the imposed magnetic field and n is the quantum number corresponding to the Landau level.

The AB flux, or **A** wave, can be generated in such two-dimensional systems and be increased by one flux quantum by changing the phase of the ground state wave function around the system. The quantized Hall effect is thus a macroscopic quantum Hall phenomenon related to the fundamental role of the phase and the $\mathbf{A}_\mu$ potential in quantum mechanics.

An important feature of the quantized Hall effect is the lack of dependence of quantization (integral multiples of $e^2/\hbar$) on the density of the mobile electrons in the sample tested (but, rather, on the symmetry of the charge density wave[159]*). Underlying this lack of dependence is a required gauge invariance of the $\mathbf{A}_\mu$ potential.* For example, the current around a metallic loop is equal to the derivative of the total electronic energy, U, of the system with respect to the magnetic flux through the loop, i.e. with respect to the $\mathbf{A}_\mu$ potential pointing around the loop[160]

$$I = \frac{(c/L)\partial U}{\partial \mathbf{A}}. \tag{3.6.1}$$

As this derivative is nonzero only with phase coherence around the loop, i.e. with an extended state, Eq. (3.6.1) is valid only if

$$A = \frac{n\hbar c}{eL}, \tag{3.6.2}$$

i.e. only with a gauge invariance for **A**.

With a gauge invariance defined for **A**, and with the Fermi level in a mobility gap, a vector potential increment changes the total energy, U, by forcing the filled states toward one edge of the total density of states spectrum and the wave functions are affected by a vector potential increment only through the location of their centers. Therefore, *gauge invariance of the **A** potential, being an exact symmetry, forces the addition of a flux quantum to result in only an excitation or de-excitation of the total system.*[160] Furthermore, the energy gap exists

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globally between the electrons and holes affected by such a perturbation in the way described, rather than in specific local density of states. Thus, the Fermi level lies *globally* in a gap in an extended state spectrum and there is no dependence of Hall conductivity on the density of mobile electrons.

Post[161–163] has also implied the vector potential in the conversion of a voltage–current ratio of the quantized Hall effect into a ratio of period integrals. If V is the Hall voltage observed transversely from the Hall current I, the relation is

$$\frac{V}{I} = \frac{\oint \mathbf{A}}{\oint \mathbf{G}} = Z_H = \text{quantized Hall impedance}, \tag{3.6.3}$$

where **G** defines the displacement field **D** and the magnetic field **H**. The implication is that

$$\frac{V}{I} = \frac{\int_0^T V dt}{\int_0^T I dt}, \tag{3.6.4}$$

where

$$\int_0^T V dt = \oint \mathbf{A}, \quad \text{the quantization of magnetic flux,} \tag{3.6.5}$$

$$\int_0^T I dt = \oint \mathbf{G}, \quad \text{the quantization of electric flux,} \tag{3.6.6}$$

and T is the cyclotron period.

Aoki and Ando[164] also attribute the universal nature of the quantum Hall effect, i.e. the quantization in units of $e^2/\hbar$ at $T = 0$ for every energy level in a finite system, to a topological invariant in a mapping from the gauge field to the complex wave function. These authors assume that in the presence of external AB magnetic fluxes, the vector potential $\mathbf{A}_0$ is replaced by $\mathbf{A}_0 + \mathbf{A}$, where $\mathbf{A} = (A_x, A_y)$. In cylindrical geometry, a magnetic flux penetrates the opening of the cylinder and the vector potential is thought of as two magnetic fluxes, $(\mathbf{\Phi}_x, \mathbf{\Phi}_y) = (A_x L, A_y L)$, penetrating inside and through the opening of a torus when periodic boundary conditions are imposed in both the x and y directions for a system of size L. According to the Byers–Yang

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theorem,[165] the physical system assumes its original state when A_x or A_y increases by $\mathbf{\Phi}_0/L$, where $\mathbf{\Phi}_0 = \hbar c/e$, the magnetic flux quantum.

The next effect examined, the de Haas–van Alphen effect, also pivots on $\mathbf{A}_\mu$ potential gauge invariance.

3.7. *The de Haas–van Alphen effect*

In 1930, de Haas and van Alphen observed what turned out to be susceptibility oscillations with a changing magnetic field which were periodic with the reciprocal field. Landau showed in the same year that for a system of free electrons in a magnetic field, the motion of the electrons *parallel* to the field is *classical*, while the motion of the electrons *perpendicular* to the field is *quantized*; and Peierls showed in 1933 that this holds for free electrons in a metal (with a spherical Fermi surface). Therefore, the free energy of the system and thus the magnetic moment ($M = \partial F/\partial H$) oscillates with the magnetic field H. This oscillation is the major cause of the De Haas–Van Alphen effect.

In 1952, Onsager showed that the frequencies of oscillations are directly proportional to the extremal cross-sections of the Fermi surface perpendicular to the magnetic field. If $\mathbf{p}$ is the electronic momentum and

$$\left[p - \left(\frac{e}{c}\right)\mathbf{A}\right] \tag{3.7.1}$$

is the canonical momentum [cf. Eqs. (3.5.5)], then

$$\oint \left[p - \left(\frac{e\mathbf{A}}{c}\right) \cdot dl\right] = (n + \gamma)h, \tag{3.7.2}$$

where n is an integer and γ is a phase factor. The relation of the $\mathbf{A}$ vector potential and the real space orbit is

$$\oint \mathbf{A} \cdot dl = \int \nabla \times \mathbf{A} \cdot dS = \mathbf{H}S, \tag{3.7.3}$$

where S is the area of the orbit in real space. Furthermore, electron paths in momentum space have the same shape as those in real space but changed in scale and turned through 90°, due to the Lorentz force relation: $d\mathbf{p}/dt = (e/c)(\nabla \times \mathbf{H})$.

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Therefore, as (i) the area of the orbit in momentum space is $\boldsymbol{S} = (n + \gamma)(e\hbar H/c)$ and (ii) the susceptibility is $-(1/H)(\partial F/\partial H)$, which is periodic in $1/H$ with period $\Delta(1/H) = 2\pi e/c\hbar \boldsymbol{S}$, there is a direct influence of the **A** vector potential on the de Haas–van Alphen effect due to the phase factor dependence [Eq. (3.7.2)]. Thus the validation of Eqs. (3.7.1) and (3.7.2) requires $\mathbf{A}_\mu$ potential gauge invariance. (The relation between the AB effect and the quantized Hall effect has been observed by Timp *et al.*[71])

Two effects have now been examined pivoting on $\mathbf{A}_\mu$ potential gauge invariance. *This gauge invariance implies flux conservation, i.e. a global conservation law*. The next effect examined, the Sagnac effect, makes explicit the consequences of this global conservation.

3.8. *The Sagnac effect*

In 1913, Sagnac demonstrated a fringe shift by rotating an interferometer (with a polygonal interference loop traversed in opposite senses) at high speed[166–168] (Fig. 3.8.1). Einstein's general theory of relativity predicts a phase shift proportional to the angular velocity and to the area enclosed by the light path — not because the velocity of the two

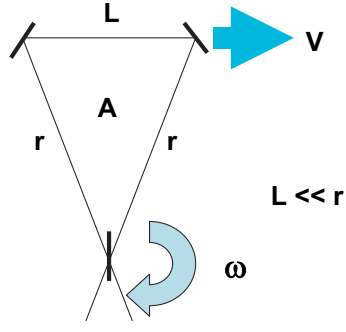

Fig. 3.8.1. The Sagnac interferometer in which the center of rotation coincides with the beam splitter location. The Sagnac phase shift is independent of the location of the center of rotation and the shape of the area. The phase shift along L is independent of r. (After Silvertooth.[169])

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beams is different, but because they each have their own time. However, *the AB, AAS and topological phase effects deny Lorentz invariance to the electromagnetic field as any field's natural and inevitable implication, i.e. Lorentz invariance is not "built in" to the Maxwell theory — it is a gauge implied by special* $\boldsymbol{A}_\mu$ *potential conditions, i.e. special boundary conditions imposed on the electromagnetic field.* Therefore, the Einstein interpretation pivots on unproven boundary conditions and the effect is open to other, competing, explanations (cf. Ref. 170).

A different explanation is offered by the Michelson *et al.*[171–173] experiments of 1924–1925. These investigators predicted a phase shift more simply on the basis of a difference in the velocity of the counter propagating beams and the earth rotating in a stationary ether without entrainment. (It should be noted that the beam path in the well-known Michelson–Morley 1886 interferometer[174] does not enclose a finite surface area. Therefore this experiment cannot be compared with the experiments and effects examined in the present review, and in fact, according to these more recent experiments, no fringe shift can be expected as an outcome of a Michelson–Morley experiment, i.e, the experiment was not a test for the presence of an ether.[h])

Post[175] argues that the Sagnac effect demonstrates that the space–time formulations of the Maxwell equations do not make explicit the constitutive properties of free space. The identification $\mathbf{E} = \mathbf{D}, \mathbf{H} = \mathbf{B}$, in the absence of material polarization mechanisms in free space, is the so called *Gaussian field identification*.[176] This identification is equivalent to an unjustified adoption of Lorentz invariance. However, the Sagnac effect and the well-used ring laser gyro on which it is based indicate that in a rotating frame, the Gaussian identity does not apply. This requirement of metric independence was proposed by Van Dantzig.[177] In order to define the constitutive relations between the fields **E** and **B**

[h]The author is indebted to an anonymous reviewer for indicating the paper by Post[181] which shows that not only does the Michelson and Morley experiment not disprove the existence of an ether, but that the experiment should give a null result regardless of whether the motion is uniform or not. Post also demonstrates a mutual relation between the Michelson–Morley and Sagnac experiments.

constituting a covariant six-vector $\mathbf{F}_{\lambda\nu}$, and the fields **D** and **H**, constituting a contravariant six-vector, $\mathbf{G}^{\lambda\nu}$, the algebraic relation[176]

$$\mathbf{G}^{\lambda\nu} = \frac{1}{2}\chi^{\lambda\nu\sigma\kappa}\mathbf{F}_{\lambda\nu} \tag{3.8.1}$$

was proposed, where $\chi^{\lambda\nu\sigma\kappa}$ is the constitutive tensor and Eq. (3.8.1) is the constitutive map. The generally invariant vector d'Alembertian (wave equation) is

$$\partial_\nu \chi^{\lambda\nu\sigma\kappa}\partial_\sigma \mathbf{A}_\kappa = 0, \tag{3.8.2}$$

indicating the vector potential dependence.

The pivotal role of the vector potential is due to the flux conservation, which is a *global* conservation law.[161,178] The *local* conservation law of flux

$$d\mathbf{F} = 0, \tag{3.8.3}$$

excludes a role for the **A** potential (**F** is inexact). However, only if

$$\oint \mathbf{F} = 0 \tag{3.8.4}$$

is it possible to state that $d\mathbf{A} = \mathbf{F}$ (**F** is exact). In other words, $d\mathbf{F} = 0$ implies $\oint \mathbf{F} = 0$ *only if the manifold over which* ***F*** *is defined is compact and simply connected*, e.g. one-connectedness (contractable circles), two-connectedness (contractable spheres) and three-connectedness (contractable three-spheres).

Post[179–181] argues that the constitutive relations of the medium-free fields **E** and **H** to the medium left out treatment of free space as a "medium." If **C** is the differential three-form of charge and current density, then the *local* conservation of charge is expressed by

$$d\mathbf{C} = 0, \tag{3.8.5}$$

and the *global* definition is

$$\mathbf{C} = d\mathbf{G}. \tag{3.8.6}$$

The Post relation is in accord with the symmetry of space–time and momentum–energy required by the reciprocity theory of Born[182] and, more recently, that of Ali.[183,184] Placing these issues in a larger

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context, Hayden[185] has argued that the classical interpretation of the Sagnac effect indicates that the speed of light is not constant. Furthermore, Hayden,[186] emphasizing the distinction between kinematics (which concerns space and time) and mechanics (which concerns mass, energy, force and momentum), argues that time dilation (a kinematic issue) is difficult to distinguish from changes in mass (a mechanics issue) owing to the way time is measured. The demonstration of an analogy between Josephson-like oscillations and the Sagnac effect[254] supports this viewpoint.

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3.9. *Summary*

In summary, the following effects have been examined:

(i) The Aharonov–Bohm and Altshuler–Aronov–Spival effects, in which changes in the $\mathbf{A}_\mu$ potential at a third location indicate differences in the $\mathbf{A}_\mu$ field along *two* trajectories at *two* other locations.
(ii) The topological phase effects, in which changes in the spin direction or polarization defined by the $\mathbf{A}_\mu$ potential at one location, *a*, are different from that at another location, *b*, owing to topological winding of the trajectory between the *two* locations *a* and *b*.
(iii) Stokes' theorem, which requires precise boundary conditions for *two* fields — the local and global fields — for exact definition in terms of the $\mathbf{A}_\mu$ potential.
(iv) Ehrenberg and Siday's derivation of the refractive index, which describes propagation between *two* points in a medium and which requires gauge invariance of the $\mathbf{A}_\mu$ potential.
(v) The Josephson effect, which implies the $\mathbf{A}_\mu$ potential as a local-to-global operator connecting *two* fields.
(vi) The quantized Hall effect, which requires gauge invariance of the $\mathbf{A}_\mu$ potential in the presence of *two* fields.
(vii) The de Haas–van Alphen effect, which requires gauge invariance of the $\mathbf{A}_\mu$ potential in the presence of *two* fields.

(viii) The Sagnac effect, which requires flux conservation, i.e. gauge invariance of the $\mathbf{A}_\mu$ potential in comparing *two* fields before and after movement.

All these effects pivot on a physical definition of $\mathbf{A}_\mu$ potentials. In the next section, the theoretical reasons for questioning the completeness of Maxwell's theory are examined as well as the reasons for the physical effectiveness of the $\mathbf{A}_\mu$ potentials in the presence of two fields. The $\mathbf{A}_\mu$ potentials have an ontology or physical meaning as *local* operators mapping *local* e.m. fields onto *global* spatiotemporal conditions. This operation is measurable if there is a second comparative mapping of the local conditioned fields in a many-to-one fashion (multiple connection).

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4. Theoretical Reasons for Questioning the Completeness of Maxwell's Theory

Yang[27,98,187,188] interpreted the electromagnetic field in terms of a nonintegrable (i.e. path-dependent) phase factor by an examination of Dirac's monopole field.[189,190] According to this interpretation, the AB effect is due to the existence of this phase factor whose origin is due to the topology of connections on a fiber bundle.

The *phase factor*

$$\exp\left[\frac{ie}{\hbar c}\oint \mathbf{A}_\mu dx^\mu\right], \tag{4.1}$$

according to this view, is physically meaningful, but not the *phase*

$$\left(\frac{ie}{\hbar c}\oint \mathbf{A}_\mu dx^\mu\right), \tag{4.2}$$

which is ambiguous because different phases in a region may describe the same physical situation. The phase factor, on the other hand, can distinguish different physical situations having the same field strength but different action. Referring to Fig. 4.1, the phase factor for any

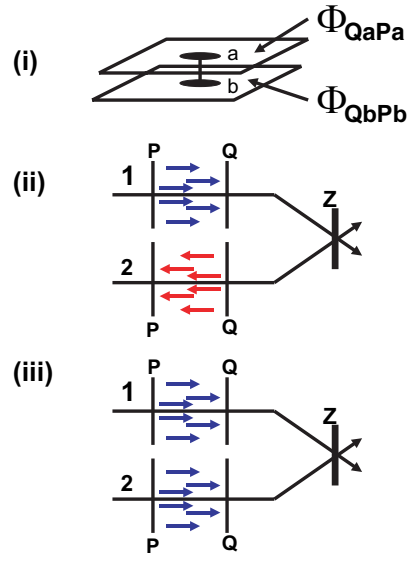

Fig. 4.1. (i) The overlap area [Z in (ii) and (iii)] showing a mapping from location a to b. The phase factor Φ_{QaPa} is associated with the e.m. field which arrived at Z through path 1 in (ii) and (iii) and Φ_{QbPb} with the e.m. field which arrived at Z through path 2 in (ii) and (iii). (ii) In paths 1 and 2 the e.m. fields are conditioned by an **A** field between P and Q oriented in the direction indicated by the arrows. Note the reversal in direction of the **A** field in paths 1 and 2, hence $S(\mathrm{P}) \neq S(\mathrm{Q})$ and $\Phi_{QaPa} \neq \Phi_{QbPb}$. (iii) Here the conditioning **A** fields are oriented in the same direction, hence there is no noticeable gauge transformation and no difference noticeable in the phase factors $S(\mathrm{P}) = S(\mathrm{Q})$ and $\Phi_{QaPa} = \Phi_{QbPb}$. (After Ref. 98.)

path from, say, P to Q, is

$$\Phi_{QP} = \exp\left[\frac{ie}{\hbar c}\int_{Q}^{P} \mathbf{A}_{\mu} dx^{\mu}\right]. \tag{4.3}$$

For a *static magnetic monopole* at an origin defined by the spherical coordinate, $r = 0, \theta$ with azimuthal angle ϕ, and considering the region R of all space–time *other than this origin*, the gauge transformation in the overlap of two regions, a and b, is

$$S_{ab} = \exp(-i\alpha) = \exp\left[\left(\frac{2ige}{\hbar c}\right)\phi\right], \quad 0 \leq \phi \leq 2\pi, \tag{4.4}$$

where g is the monopole strength.

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This is an allowed gauge transformation if and only if

$$\frac{2ge}{\hbar c} = \text{an integer} = D, \tag{4.5}$$

which is Dirac's quantization. Therefore

$$S_{\text{ab}} = \exp(iD\phi). \tag{4.6}$$

In the overlapping region, there are two possible phase factors, Φ_{QaPa} and Φ_{QbPb}, and

$$\Phi_{\text{QaPa}} S(P) = S(Q)\Phi_{\text{QbPb}}, \tag{4.7}$$

a relation which states that $(\mathbf{A}_\mu)_{\text{a}}$ and $(\mathbf{A}_\mu)_{\text{b}}$ are related by a gauge transformation factor.

The general implication is that for a gauge with any field defined on it, the total magnetic flux through a sphere around the origin $r = 0$ is independent of the gauge field and only depends on the gauge (phase)

$$\int\int \phi_{\mu\nu} dx^\mu dx^\nu = \left(\frac{-i\hbar c}{e}\right) \int \frac{\partial}{\partial x^\mu} (\ln S_{\text{ab}}) dx^\mu, \tag{4.8}$$

where the integral is taken around any loop around the origin $r = 0$ in the overlap between the R_a and R_b, as, for example, in an equation for a sphere $r = 1$. As S_{ab} is single-valued, this integral must be equal to an integral multiple of a constant (in this case 2πi).

Another implication is that if the $\mathbf{A}_\mu$ potentials originating from, or passing through, two or more different local positions are gauge-invariant when compared to another, again different, local position, then the referent providing the basis or metric for the comparison of the phase differences at this local position is a unit magnetic monopole. The unit monopole, defined at $r = 0$, is unique in not having any internal degrees of freedom.[191] Furthermore, both the monopole and charges are topologically conserved, but whereas *electric* charge is topologically conserved in U(1) symmetry, *magnetic* charge is only conserved in SU(2) symmetry.

Usually, there is no need to invoke the monopole concept as the $\mathbf{A}_\mu$ field is, as emphasized here, treated as a mathematical, not physical, construct in contemporary classical physics. However, in quantum

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physics, the wave function satisfies a partial differential equation coupled to boundary conditions. The boundary conditions in the doubly connected region outside of the solenoid volume in an AB experiment result in the single-valuedness of the wave function, which is the reason for quantization. Usually, for example in textbooks explaining the theory of electromagnetism as noted above, Stokes' theorem is written as

$$\oint \mathbf{A}dx = \int\int \mathbf{H} \cdot ds = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n}\, da, \tag{4.9}$$

and no account is taken of space–time overlap of regions with fields derived from different sources having undergone different spatiotemporal conditioning, and no boundary conditions are taken into account. Therefore, no quantization is required.

There is no lack of competing opinions on what the theoretical basis is for the magnetic monopole implied by gauge-invariant $\mathbf{A}_\mu$ potentials[145,192–195] (cf. Ref. 196 for a guide to the literature). The Dirac magnetic monopole is an anomalously shaped (string) magnetic dipole at a singularity.[189,190] The Schwinger magnetic monopole is essentially a double singularity line.[197,198] However, gauge-invariant $\mathbf{A}_\mu$ potentials are the local manifestation of global constructs. This precludes the existence of *isolated* magnetic monopoles, but permits them to exist *globally* in any situation with the requisite energy conditioning. Wu and Yang,[98,199,200] 't Hooft,[201,202] Polyakov[203] and Prasad and Sommerfeld[204] have described such situations.

More recently, Zeleny[205] has shown that Maxwell's equations and the Lorentz force law can be derived, not by using invariance of the action (Hamilton's principle) or by using constants of the motion (Lagrange's equations), but by considerations of symmetry. If, in the derivation, the classical **A** field is dispensed with in favor of the electromagnetic tensor **F**, classical magnetic monopoles are obtained, which are without strings and can be extended particles. Such particles are accelerated by a magnetic field and bent by an electric field.

Related to mechanisms of monopole generation is the Higgs field (Φ) approach to the vacuum state.[206–208] The field, in some scenarios,

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breaks a higher order symmetry field, e.g. SU(2), **G**, into **H** of U(1) form. The **H** field is then proportional to the electric charge.

There are at least five types of monopoles presently under consideration:

(1) The Dirac monopole, a point singularity with a string source. The $\mathbf{A}_\mu$ field is defined everywhere except on a line joining the origin to infinity, which is occupied by an infinitely long solenoid, so that $\mathbf{B} = \nabla \cdot \mathbf{A}$ (a condition for the existence of a magnetic monopole). Dirac's approach assumes that a particle has either electric or magnetic charge but not both.
(2) Schwinger's approach, on the other hand, permits the consideration of particles with both electric and magnetic charge, i.e. dyons.[197,198]
(3) The 't Hooft–Polyakov monopole, which has a smooth internal structure but without the need for an external source. There is, however, the requirement for a Higgs field.[206–208] The 't Hooft–Polyakov model can be put in the Dirac form by a gauge transformation.[145]
(4) The Bogomol'nyi–Prasad–Sommerfeld monopole is one in which the Higgs field is massless, long range and with a force which is always attractive.
(5) The Wu–Yang monopole requires no Higgs field, has no internal structure and is located at the origin. It requires multiply connected fields. The singular string of the Dirac monopole can be moved arbitrarily by a gauge transformation.[209] Therefore, the Dirac and the Wu–Yang monopoles can be made compatible. The Higgs field formalism can also be related to that of Wu–Yang, in which only the exact symmetry group appears.

Goddard and Olive[145] demonstrated that there are two conserved currents for a monopole solution: the usual Noether current, whose conservation depends on the equations of motion; and a topological current, whose conservation is independent of the equations of motion.

Yang[210] showed that if space–time is divided into two overlapping regions in both of which there is a vector potential **A** with gauge transformation between them in the overlap regions, then the proper

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definition of Stokes' theorem when the path integral goes from region 1 to another 2 is[199,200]

$$\int_{A}^{C} \mathbf{A}dx = \int_{A}^{B} \mathbf{A}_1 dx = \int_{B}^{C} \mathbf{A}_2 dx + \beta(B). \tag{4.10}$$

The β function is defined by the observation that in the region of overlap, the difference of the vector potentials $\mathbf{A}_1 - \mathbf{A}_2$ is curl-less as the two potentials give the same local electromagnetic field.

There are also general implications. Gates[211] takes the position that all the fundamental forces in nature arise as an expression of gauge invariance. If a phase angle $\theta(x, t) = -(i/2)\ln[\psi/\hat{\psi}]$ is defined for quantum-mechanical systems, then although the difference $\theta(x_1, t) - \theta(x_2, t)$ is a *gauge-dependent* quantity, the expression

$$\theta(x_1, t) - \theta(x_2, t) + \left(\frac{e}{\hbar c}\right) \int_{x_1}^{x_2} ds\, \mathbf{A}(s, t) \tag{4.11}$$

is *gauge-invariant*. (Note that according to the Wu–Yang interpretation, the last expression should be $\exp[(e_0/\hbar c) \int_{x_1}^{x_2} ds\, \mathbf{A}(s, t)]$. Therefore, any measurable quantity which is a function of such a difference in phase angles must also depend on the vector potentials shown. Setting the expression (4.11) to zero gives a general description of both the AB and Josephson effects. Substituting $\exp[(e_0/\hbar c) \int_{x_1}^{x_2} ds\, \mathbf{A}(s, t)]$ for the final term gives a description of the topological phase effect.

The phenomena described above are a sampling of a range of effects. There are probably many yet to be discovered, or provided with the description of an "effect." A unifying theme of all of them is that the physical effect of the $\mathbf{A}_\mu$ potentials is only describable (a) when *two* or more fields undergo different spatiotemporal conditioning *and* there is also a possibility of cross-comparison (many-to-one mapping) or, equivalently, (b) in the situation of a field trajectory with a *beginning* (giving the field before the spatiotemporal conditioning) and an *end* (giving the field after the conditioning), *and* again a possibility of cross-comparison. Setting boundary conditions to an e.m. field gives gauge invariance but without necessarily providing the conditions for detection of the gauge invariance. The gauge-invariant $\mathbf{A}_\mu$

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potential field operates on an e.m. field state to an extent determined by global symmetries defined by spatiotemporal conditions, but the effect of this operation or conditioning is only detectable under the global conditions (a) and (b). With no interfield mapping or comparison, as in the case of the solitary electromagnetic field, the $\mathbf{A}_\mu$ fields remain ambiguous, but this situation occurs only if no boundary conditions are defined — an ambiguous situation even for the electric and magnetic fields. Therefore, the $\mathbf{A}_\mu$ potentials in all useful situations have a meaningful physical existence related to boundary condition choice — even when no situation exists for their comparative detection. *What* is *different between the* $\mathbf{A}_\mu$ *field and the electric and magnetic fields is that the ontology of the* $\mathbf{A}_\mu$ *potentials is related to the global spatiotemporal boundary conditions in a way in which the local electric and magnetic fields are not*. Owing to this global spatiotemporal (boundary condition) dependence, the operation of the $\mathbf{A}_\mu$ potentials is a one-to-many, local-to-global mapping of individual e.m. fields, the nature of which is examined in Subsec. 5.2. The detection of such mappings is only within the context of a *second* comparative projection, but this time global-to-local.

This section has addressed theoretical reasons for questioning the completness of U(1) symmetry, or Abelian Maxwell theory in the presence of *two* local fields separated globally. In the next section, a pragmatic reason is offered: propagating velocities of e.m. fields in lossy media cannot be calculated in U(1) Maxwell theory. The theoretical justification for physically defined $\mathbf{A}_\mu$ potentials lies in the application of Yang–Mills theory — not to high energy fields, where the theory first found application, but to low energy fields crafted to a specific group of transformation rules by boundary conditions. This is a new application of Yang–Mills theory.

5. Pragmatic Reasons for Questioning the Completeness of Maxwell's Theory

5.1. *Harmuth's ansatz*

A satisfactory concept permitting the prediction of the propagation velocity of e.m. signals does not exist within the framework

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of Maxwell's theory[212–214] (see also Refs. 215–223). The calculated group velocity fails for two reasons: (i) it is almost always larger than the speed of light for RF transmission through the atmosphere; (ii) its derivation implies a transmission rate of information equal to zero. Maxwell's equations also do not permit the calculation of the propagation velocity of signals with bandwidth propagating in a lossy medium, and all the published solutions for propagation velocities assume sinusoidal (linear) signals.

In order to remedy this state of affairs, Harmuth proposed an amendment to Maxwell's equations, which I have called the *Harmuth ansatz*.[224–226] The proposed amended equations (in Gaussian form) are

Coulomb's law [Eq. (1.1)]:

$$\nabla \cdot D = 4\pi\rho_e; \tag{5.1.1}$$

Maxwell's generalization of Ampère's law [Eq. (1.2)]:

$$\nabla \times \mathbf{H} = \left(\frac{4\pi}{c}\right)\mathbf{J}_e + \left(\frac{1}{c}\right)\frac{\partial \mathbf{D}}{\partial t}; \tag{5.1.2}$$

Postulate of the presence of free magnetic poles:

$$\nabla \cdot \mathbf{B} = \rho_m; \tag{5.1.3}$$

Faraday's law with magnetic monopole:

$$\nabla \times \mathbf{E} + \left(\frac{1}{c}\right)\frac{\partial \mathbf{B}}{\partial t} + \left(\frac{4\pi}{c}\right)\mathbf{J}_m = 0; \tag{5.1.4}$$

and the constitutive relations:

$$\mathbf{D} = \varepsilon\mathbf{E}, \tag{1.5 and 5.15}$$

$$\mathbf{B} = \mu\mathbf{H}, \tag{1.7 and 5.15}$$

$$\mathbf{J}_e = \sigma\mathbf{E} \text{ (electric Ohm's law)}, \tag{1.6 and 5.15}$$

$$\mathbf{J}_m = s\mathbf{E} \text{ (magnetic Ohm's law)}, \tag{5.1.8}$$

where $\mathbf{J}_e$ is electric current density, $\mathbf{J}_m$ is magnetic current density, ρ_e is electric charge density, ρ_m is magnetic charge density, σ is electric conductivity and s is magnetic conductivity.

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Setting $\rho_e = \rho_m = \nabla \cdot \mathbf{D} = \nabla \cdot \mathbf{B} = 0$ for free space propagation gives

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\varepsilon \partial \mathbf{E}}{\partial t}, \tag{5.1.9}$$

$$\nabla \times \mathbf{E} + \frac{\mu \partial \mathbf{H}}{\partial t} + s\mathbf{H} = 0 \tag{5.1.10}$$

$$\varepsilon \nabla \cdot \mathbf{B} = \mu \nabla \cdot \mathbf{B} = 0, \tag{5.1.11}$$

and the equations of motion

$$\frac{\partial E}{\partial y} + \frac{\mu \partial H}{\partial t} + sH = 0, \tag{5.1.12}$$

$$\frac{\partial H}{\partial y} + \frac{\varepsilon \partial E}{\partial t} + \sigma E = 0. \tag{5.1.13}$$

Differentiating Eqs. (5.1.12) and (5.1.13) with respect to y and t permits the elimination of the magnetic field, resulting in [Ref. 212, Eq. (21)]

$$\frac{\partial^2 E}{\partial y^2} - \frac{\mu \varepsilon \partial^2 E}{\partial t^2} - (\mu\sigma + \varepsilon s)\frac{\partial E}{\partial t} - s\sigma E = 0, \tag{5.1.14}$$

which is a two-dimensional Klein–Gordon equation (without boundary conditions) in the sine-Gordon form

$$\frac{\partial^2 E}{\partial y^2} - \left(\frac{1}{c^2}\right)\frac{\partial^2 E}{\partial t^2} - \alpha \sin(\beta E(y,t)) = 0, \tag{5.1.15}$$

$$\alpha \sin(\beta E(y,t)) \approx -\left[\frac{\alpha\beta E}{\partial t} - O(E)\right], \tag{5.1.16}$$

$$\alpha = \exp(+\mu\sigma) \tag{5.1.17}$$

$$\beta = \exp(+\varepsilon\sigma)\beta, \tag{5.1.18}$$

where $(\partial^2 E/\partial y^2 - (1/c^2)\partial^2 E/\partial t^2)$is the "nonlinear" term and $(\alpha \sin(\beta E(y,t)))$ is the dispersion term. This match of "nonlinearity" and dispersion permits soliton solutions, and the field described by Eq. (5.1.15) has a "mass," $m = \sqrt{\alpha\beta}$.

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Equation (5.1.15) may be derived from the Lagrangian density

$$L = \frac{1}{2}\left[\left(\frac{\partial E}{\partial y}\right)^2 - \left(\frac{\partial E}{\partial t}\right)^2\right] - V(E), \tag{5.1.19}$$

where

$$V(E) = \frac{\alpha}{\beta}(1 - \cos \beta E). \tag{5.1.20}$$

The wave equation for E has a solution which can be written in the form

$$E(y, t) = E_E(y, t) = E_0[w(y, t) + F(y)], \tag{5.1.21}$$

where $F(y)$ indicates that an electric step function is the excitation.

A wave equation for $F(y)$ is

$$\frac{d^2F}{dy^2} - s\sigma F = 0, \tag{5.1.22}$$

with the solution

$$F(y) = A_{00} \exp[-yL] + A_{01} \exp\left[\frac{y}{L}\right], \quad L = (s\sigma)^{-1/2}. \tag{5.1.23}$$

Boundary conditions require $A_{01} = 0$ and $A_{00} = 1$, therefore

$$F(y) = \exp\left[-\frac{y}{L}\right]. \tag{5.1.24}$$

Insertion of Eq. (5.1.21) into Eq. (5.1.14) gives [Ref. 212, Eq. (40)]

$$\frac{\partial^2 w}{\partial y^2} - \frac{\mu\varepsilon\partial^2 w}{\partial t^2} - (\mu\sigma + \varepsilon s)\frac{\partial w}{\partial t} - s\sigma w = 0, \tag{5.1.25}$$

which we can again put into sine-Gordon form

$$\frac{\partial^2 w}{\partial y^2} - \left(\frac{1}{c^2}\right)\frac{\partial^2 w}{\partial t^2} - \alpha \sin(\beta w(y, t) = 0. \tag{5.1.26}$$

Harmuth[212] developed a solution to (5.1.21) by seeking a general solution of $w(y, t)$ using a separation-of-variables method (and setting s to zero after a solution is found). This solution works well, but

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we now indicate another solution. The solutions to the sine-Gordon equation (5.1.15) are the hyperbolic tangents

$$E(y) = \pm\left(\frac{8\sqrt{\alpha\beta}}{\beta^2}\right)\tan^{-1}\left(\exp\left[\frac{y - y_0 - ct}{\sqrt{1-c^2}}\right]\right), \qquad (5.1.27)$$

where $c = \sqrt{1/\mu\varepsilon}$, which describe solitons. It is also well known that the sine-Gordon and Thirring[256]models are equivalent[145] and that both admit two currents: one a Noether current and the other a topological current.

The following remarks may now be made: the introduction of $F(y) = \exp[-y/L]$, according to the Harmuth ansatz (Ref. 212, p. 253), provides integrability. It is well known that soliton solutions require complete integrability. According to the present view, $F(y)$ also provides the problem with boundary conditions, the *necessary condition* for $\mathbf{A}_\mu$ potential invariance. Equation (5.1.24) is, in fact, a phase factor [Eqs. (4.1), (4.3), (4.4) and (4.6)]. Furthermore, Eq. (5.1.21) is of the form of Eq. (4.11). Therefore the Harmuth ansatz amounts to a definition of boundary conditions — i.e. obtains the condition of separate electromagnetic field comparison by overlapping fields — *which permits complete integrability and soliton solutions to the extended Maxwell equations*. Furthermore, it was already seen, above, that with boundary conditions defined, the $\mathbf{A}_\mu$ potentials are gauge-invariant, implying a magnetic monopole and charge. It is also known that the magnetic monopole and charge constructs only exist under certain field symmetries. In the next section, methods are presented for conditioning fields into those higher order symmetries.

5.2. *Conditioning the electromagnetic field into altered symmetry: Stokes' interferometers and Lie algebras*

The theory of Lie algebras offers a convenient summary of the interaction of the $\mathbf{A}_\mu$ potential operators with the **E** fields.[257,258] The relevant parts of the theory are as follows. A manifold, ***L***, is a set of elements in one-to-one correspondence with the points of a vector manifold ***M***. ***M*** is a set of vectors called points of ***M***. A Lie group, ***L***, is a group which is also a manifold on which the group operations are continuous. There exists an invertible function, ***T***, which maps each point

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x in $\boldsymbol{M}$ to a group element $X = \boldsymbol{T}(x)$ in $\boldsymbol{L}$. The group $\boldsymbol{M}$ is a global parametrization of the group $\boldsymbol{L}$.

If $\partial = \partial_x$ is the derivative with respect to a point on a manifold $\boldsymbol{M}$, then the *Lie bracket* is

$$[a, b] = a \cdot \partial b - b \cdot \partial a = a\nabla \cdot b - b \cdot \nabla a, \tag{5.2.1}$$

where a and b are arbitrary vector-valued functions. Furthermore, with $\wedge$ signifying the outer product,[227,228]

$$[a, b] = \partial \cdot (a \wedge b) - b\partial \cdot a + a\partial \cdot b, \tag{5.2.2}$$

showing that the Lie bracket preserves tangency.

The fundamental theorem of Lie group theory is that the Lie bracket $[a, b]$ of differential fields on any manifold is again a vector field. A set of vector fields, $a, b, c, \ldots$, on any manifold forms a Lie algebra if it is closed under the Lie bracket and all fields satisfy the Jacobi identity:

$$[[a, b], c] + [[b, c], a] + [[c, a], b] = 0. \tag{5.2.3}$$

If $c = 0$, then

$$[a, b] = 0. \tag{5.2.4}$$

The $\mathbf{A}_\mu$ potentials effect mappings, $\boldsymbol{T}_1$, from the global field to the $\mathbf{E}$ local fields, considered as group elements in $\boldsymbol{L}$; and there must be a second mapping, $\boldsymbol{T}_2$, of those separately conditioned $\mathbf{E}$ fields now considered global, onto a single local field for the $\boldsymbol{T}_1$ mappings to be detected (measurable). That is, in the AB situation (and substituting fields for electrons), if the E fields traversing the two paths are $\mathbf{E}_1$ and $\mathbf{E}_2$, and those fields before and after interaction with the $\mathbf{A}_\mu$ field are $\mathbf{E}_{1i}$ and $\mathbf{E}_{1f}$ and $\mathbf{E}_{2i}$ and $\mathbf{E}_{2f}$ respectively, then $\mathbf{E}_{1f}+\mathbf{E}_{2f} = \boldsymbol{T}(\mathbf{E}_{1i}+\mathbf{E}_{2i})$, where $x_1 = \mathbf{E}_{1i}$ and $x_2 = \mathbf{E}_{2i}$ are points in $\boldsymbol{M}$, and $X = (\mathbf{E}_{1f} + \mathbf{E}_{2f})$ is considered a *group* point in $\boldsymbol{L}$ and $\boldsymbol{T} = \boldsymbol{T}_1 + \boldsymbol{T}_2, \boldsymbol{T}_1 = \boldsymbol{T}_1^{-1}$. In the same situation, although $(\mathbf{E}_{1f} - \mathbf{E}_{2f}) = \exp(i\hbar/e) \oint \mathbf{A}_\mu dx^\mu = \Phi$ [i.e. the phase factor detected at Z in a separate second mapping, $\boldsymbol{T}_2$, in Fig. 5.2.1 can be ascribed to a nonintegrable (path-independent) phase factor], the influence of the first, $\boldsymbol{T}_1$, mapping or conditioning of $\mathbf{E}_{1i} + \mathbf{E}_{2i}$ by the $\mathbf{A}_\mu$ operators along the separate path trajectories *preceded* that second mapping, $\boldsymbol{T}_2$, at Z. Therefore the $\mathbf{A}_\mu$ potential

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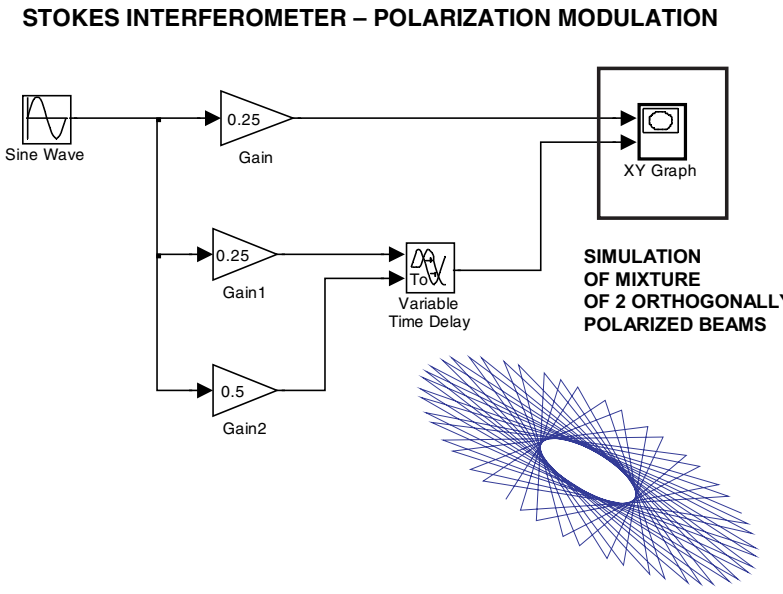

Fig. 5.2.1. Wave guide system paradigm for polarization-modulated ($\partial\phi/\partial t$) wave emission. This is a completely adiabatic system in which oscillating energy enters from the left and exits from the right. On entering from the left, energy is divided into two parts equally. One part, of amplitude $E/2$, is polarization-rotated and used in providing phase modulation, $\partial\phi/\partial t$ — this energy is spent (absorbed) by the system in achieving the phase modulation; the other part, of amplitude E/2, is divided into two parts equally, so that two oscillating wave forms of amplitude $E/4$ are formed for later superposition at the output. Owing to the phase modulation of one of them with respect to the other, $0 < \phi < 360°$, and their initial orthogonal polarization, the output is of continuously varying polarization. The choice of wave division into two equal parts is arbitrary. (From Refs. 230 and 231.)

field operators produce a mapping of the global spatiotemporal conditions onto local fields, which, in the case we are considering, are the separate $\mathbf{E}_{1i} + \mathbf{E}_{2i}$ fields. Thus, according to this conception, the $\mathbf{A}_\mu$ potentials are local operator fields mapping the local-to-global gauge ($\boldsymbol{T}_1 : \boldsymbol{M} \rightarrow L$), whose effects are detectable at a later spatiotemporal position only at an overlapping (X group) point, i.e. by a second mapping ($\boldsymbol{T}_2 : \boldsymbol{L} \rightarrow M$), permitting comparisons of the differently conditioned fields in a many-to-one (global-to-local) summation.

If $a = \mathbf{E}_{1i}$ and $b = \mathbf{E}_{2i}$, where $\mathbf{E}_{1i}$ and $\mathbf{E}_{2i}$ are local field intensities and $c = \mathbf{A}_\mu$, i.e. $\mathbf{A}_\mu$ is a local field mapping ($\boldsymbol{T}_1 : \boldsymbol{M} \rightarrow L$) according

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to gauge conditions specified by boundary conditions, then the field interactions of a, b and c, or $\mathbf{E}_{1i}$, $\mathbf{E}_{2i}$ and $\mathbf{A}_\mu$, are described by the Jacobi identity [Eq. (5.2.3)]. If $c = \mathbf{A}_\mu = 0$, then $[a, b] = 0$. With the Lorentz gauge (or boundary conditions), the $\mathbf{E}_{1i}$, $\mathbf{E}_{1f}$, $\mathbf{E}_{2i}$ and $\mathbf{E}_{2f}$ field relations are described by SU(2) symmetry. With other boundary conditions and no separate $\mathbf{A}_\mu$ conditioning, the $\mathbf{E}_{1i}$ and $\mathbf{E}_{2i}$ fields (there are no $\mathbf{E}_{1f}$ and $\mathbf{E}_{2f}$ fields in this situation) are described by U(1) symmetry relations.

The $\boldsymbol{T}_1, \boldsymbol{T}_2$ mappings can be described by classical control theory analysis and the $\mathbf{A}_\mu$ potential conditioning can be given a physical wave guide interferometer representation (cf. Ref. 229). The wave guide system considered here is completely general in that the output can be phase-, frequency- and amplitude-modulated. It is an adiabatic system (lossless) and only three of the lines are wave guides — the input, the periodically delayed line, and the output. Other lines shown are energy-expending, phase-modulating lines. The basic design is shown in Fig. 5.2.1. In this figure, the input is $E = E\exp(i\omega t)$. The output is

$$E_{\mathrm{OUT}} = \left(\frac{E}{4}\right)\exp(i\omega t) + \left(\frac{E}{4}\right)\exp(i[\omega + \exp(i\omega t) - 1]t), \quad (5.2.5)$$

where, referring to Fig 5.2.1, $\phi = F(E/2)$ and $\partial\phi/\partial t = F'(E/2)$, and it is understood that the first arm is orthogonal to the second.

The wave guide consists of two arms — the upper $(E/4)$ and the second $(E/4)$, with which the upper is combined. The lower, or third arm, merely expends energy in achieving the phase modulation of the second arm with respect to the first. This can be achieved by merely making the length of the second arm change in a sinusoidal fashion (i.e. producing a $\partial\phi/\partial t$ with respect to the first arm), or it can be achieved electro-optically. Whichever way is used, one half the total energy of the system $(E/2)$ is spent on achieving the phase modulation in the particular example shown in Fig. 5.2.1. The entropy change from input to output of the wave guide is compensated for by energy expenditure in achieving the phase modulation which causes the entropy change.

One can nest phase modulations. The next order nesting is shown in Fig. 5.2.2, and other, higher order nestings of order n, for the cases

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$\partial\phi^n/\partial t^n, n = 2, 3, \ldots$, follow the same procedure. The input is again $E = E\exp(i\omega t)$. The output is

$$E_{\mathrm{OUT}(n=2)} = \left(\frac{E}{4}\right)\exp(i\omega t) + \left(\frac{E}{4}\right)\exp(i[\phi_1 + \exp(i\phi_2 t) - 1]t), \tag{5.2.6}$$

where $\phi_1 = F_1(E/4)$, $\phi_2 = F_2(E/4)$ and $\partial\phi^2/\partial t^2 = F_1^2 \cdot F_2^2$, and again it is understood that the first arm is orthogonal to the second.

Again, the wave guide consists of two arms — the upper $(E/4)$ and the second $(E/4)$, with which the upper is recombined. The lower two arms, three and four, merely expend energy in achieving the phase modulation of the second arm with respect to the first. This again can be achieved by merely making the length of the second arm change in a sinusoidal fashion (i.e. producing a $\partial\phi^2/\partial t^2$ with respect to the first arm), or it can be achieved electro-optically for visible frequencies. Whichever way is used, one half the total energy of the system $(E/4 + E/4 = E/2)$ is spent on achieving the phase modulation of the particular sample shown in Fig. 5.2.2.

Both the systems shown in Figs. 5.2.1 and 5.2.2, and all higher orders of such systems, $\partial\phi^n/\partial t^n, n = 1, 2, 3, \ldots,$ are adiabatic with respect to the total field, and Poynting's theorem applies to them all. However, the Poynting description, or rather limiting condition, is insufficient to describe these fields exactly, and neglects the orthogonal polarization two-beam picture, and a more exact analysis is provided by the control theory picture shown here.

These wave guides we shall call Stokes interferometers. The Stokes equation is [Eq. (3.3.1)]

$$\oint \mathbf{A} \cdot dl = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n}\, da, \tag{5.2.7}$$

and the energy-expending lines of the two Stokes interferometers shown are normal to the two wave guide lines. l is varied sinusoidally,

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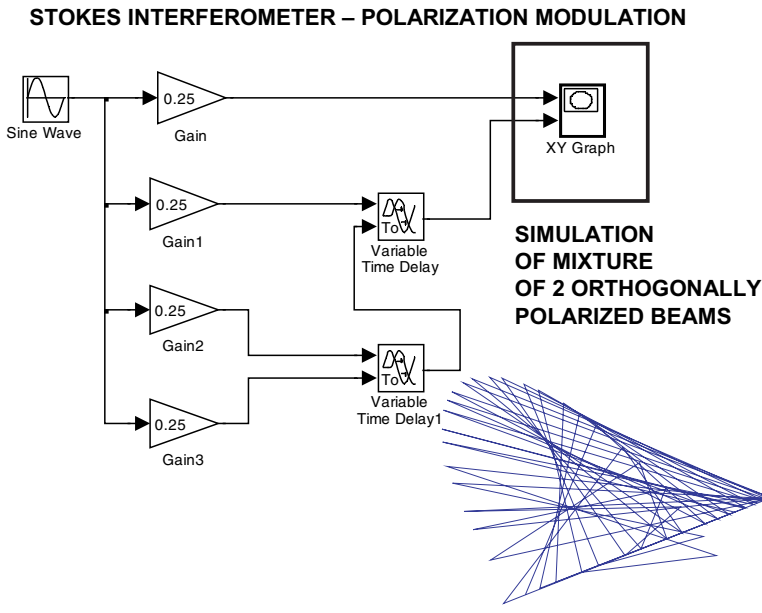

Fig. 5.2.2. Wave guide system paradigm for polarization-modulated ($\partial\phi^2/\partial t^2$) wave emission. This is a completely adiabatic system in which oscillating energy enters from the left and exits from the right. On entering from the left, the energy is divided into two parts equally. One part, of amplitude $E/2$, is used in providing phase modulation, $\partial\phi^2/\partial t^2$ — this energy is spent (absorbed) by the system in obtaining the phase modulation; the other part, of amplitude $E/2$, is divided into two parts equally, one of which is polarization-rotated, so that two oscillating wave forms of amplitude $E/4$ but initially orthogonally polarized are formed for later superposition at the output. Unlike the system shown in Fig 5.2.1, the energy expended on phase-modulating one of these waves is divided into two parts equally, of amplitude $E/4$, one of which is phase-modulated, $\partial\phi/\partial t$, with respect to the other, as in Fig. 5.2.1. The energy of the superposition of these two waves is then expended to provide a second phase-modulated $\partial\phi^2/\partial t^2$ wave which is superposed with the nondelayed wave. Owing to the phase modulation of one of them with respect to the other, $0 < \phi < 360°$, and their initial orthogonal polarization, the output is of continuously varying polarization. The choice of wave division into two equal parts is arbitrary. (From Refs. 230 and 231.)

so we have

$$\oint A \sin \omega t \, dl = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n} \, da = E_{\text{OUT}\, n=1} \quad \text{(Fig. 5.2.1)} \quad (5.2.8)$$

$$\oint A \sin \omega t \, dl = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n} \, da = E_{\text{OUT}\, n=2} \quad \text{(Fig. 5.2.2)} \quad (5.2.9)$$

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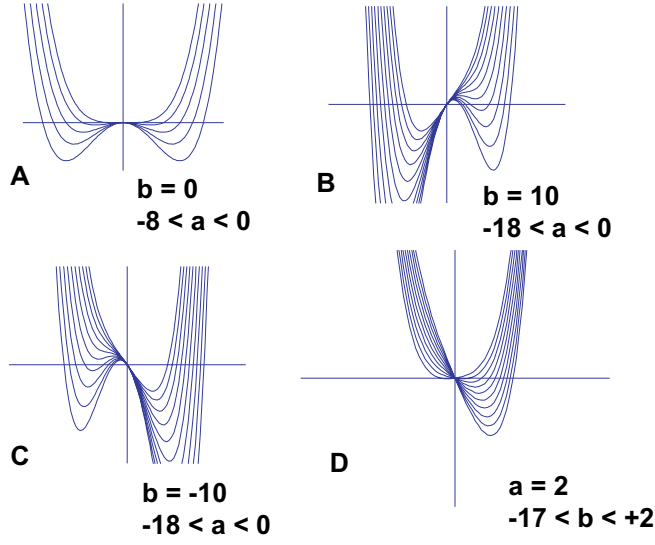

Fig. 5.2.3. Plots of $A = 1/4x^4 + 1/2ax^2 + bx + c$. (A) $b = 0$ and various values of a; (B) $b = 10$ and various values of a; (C) $b = -10$ and various values of a; (D) $a = 2$ and various values of b. For positive values of a, SU(2) symmetry is restored. For negative values of a, symmetry is broken and U(1) symmetry is obtained. (From Ref. 232.)

The gauge symmetry consequences of this conditioning are shown in Figs. 5.2.3 and 5.2.4. The potential, $\mathbf{A}_\mu$, in Taylor expansion along one coordinate is

$$A = \frac{1}{4}x^4 + \frac{1}{2}ax^2 + bx + c, \tag{5.2.10}$$

with $b < 0$ in the case of E_{in} and $b > 0$ in the case of E_{OUT}. A Stokes interferometer permits the E field to restore a symmetry which was broken before this conditioning. Thus the E_{in} field is in U(1) symmetry form and the E_{OUT} field is conditioned to be in SU(2) symmetry form. The conditioning of the **E** field to SU(2) symmetry form is the opposite of symmetry breaking. It is well known that the Maxwell theory is in U(1) symmetry form and the theoretical constructs of the magnetic monopole and charge exist in SU(2) symmetry form.[225,226,230–234]

Other interferometric methods besides Stokes interferometer polarization modulators which restore symmetry are cavity wave

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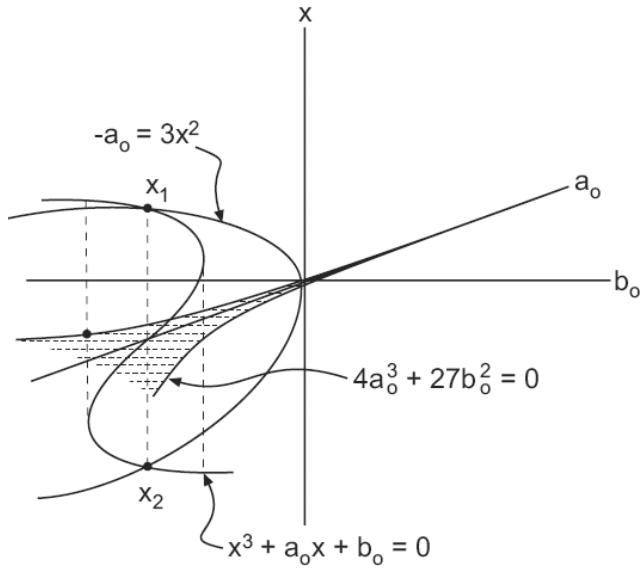

Fig. 5.2.4. A representative system defined in the (x, a, b) space. Other systems can be represented in the cusp area at other values of x, a and b. As in Fig. 5.2.3, for positive values of a, SU(2) symmetry is restored. For negative values of a, symmetry is broken and U(1) symmetry is obtained. (From Ref. 232.)

guide interferometers. For example, the Mach–Zehnder and Fabry–Perot interferometers are SU(2) conditioning interferometers[235] (Fig. 5.2.5). The SU(2) group characterizes passive lossless devices with two inputs and two outputs with the boson commutation relations

$$[E_{1^*}, E_{2^*}] = [E_{1^*}^{\Uparrow}, E_{2^*}^{\Uparrow}] = 0, \tag{5.2.11}$$

$$[E_{1^*}, E_{2^*}^{\Uparrow}] = d_{12^*}, \tag{5.2.12}$$

where $E^{\Uparrow}$ is the Hermitian conjugate of E and * signifies both *in* (entering) and *out* (exiting) fields, i.e. before and after $\mathbf{A}_\mu$ conditioning. The Hermitian operators are

$$\mathbf{J}_x = \frac{1}{2}(E_1^{\Uparrow} E_{2^*} + E_{1^*} E_2^{\Uparrow}) = \frac{1}{2}(A_1 \times B_{1\mathrm{IN}} + B_{2\mathrm{IN}} \times A_2)$$
$$= (\mathbf{A} \times \mathbf{B} - \mathbf{B} \times \mathbf{A}), \tag{5.2.13a}$$

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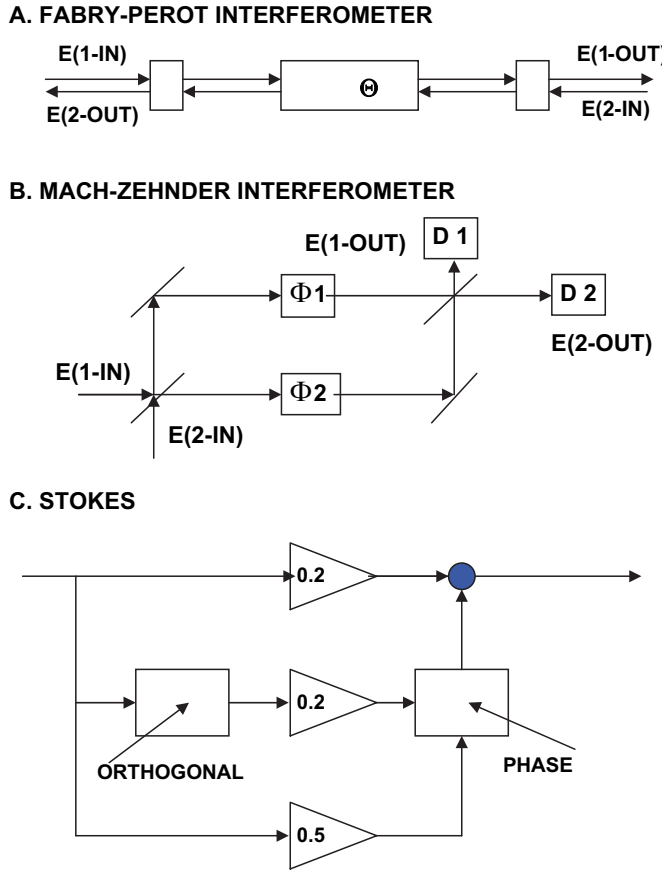

Fig. 5.2.5. SU(2) field conditioning interferometers: (A) Fabry–Perot; (B) Mach-Zehnder; (C) Stokes. (After Ref. 225.)

$$\mathbf{J}_y = -\frac{i}{2}(E_1^{\Uparrow} E_{2^*} - E_{1^*} E_2^{\Uparrow}) = -\frac{i}{2}(A_1 \times B_{1\mathrm{IN}} + B_{2\mathrm{IN}} \times A_2)$$
$$= (\mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A}), \tag{5.2.13b}$$
$$\mathbf{J}_z = \frac{1}{2}(E_1^{\Uparrow} E_{1^*} - E_{2^*} E_2^{\Uparrow}) = \frac{1}{2}(A_1 \times E_{1\mathrm{OUT}} - E_{2\mathrm{OUT}} \times A_2)$$
$$= (\mathbf{A} \times \mathbf{E} - \mathrm{E} \times \mathbf{A}), \tag{5.2.13c}$$
$$i\mathbf{J}_z = \frac{1}{2}(E_1^{\Uparrow} E_{1^*} + E_{2^*} E_2^{\Uparrow}) = -\frac{i}{2}(A_1 \times E_{1\mathrm{OUT}} + E_{2\mathrm{OUT}} \times A_2)$$
$$= (\mathbf{A} \times \mathbf{E} + \mathrm{E} \times \mathbf{A}), \tag{5.2.13d}$$

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where the substitutions are

$$\begin{aligned} E_{1^*} &= B_{2\mathrm{IN}} \text{ and } \times E_{1\mathrm{OUT}}, \\ E_{2^*} &= \times B_{1\mathrm{IN}} \text{ and } E_{2\mathrm{OUT}}, \\ E_1^{\Uparrow} &= A_1, \\ E_2^{\Uparrow} &= \times A_2, \end{aligned} \tag{5.2.14}$$

satisfying the Lie algebra

$$\begin{aligned} [\mathbf{J}_x, \mathbf{J}_y] &= i\mathbf{J}_z, \\ [\mathbf{J}_y, \mathbf{J}_z] &= i\mathbf{J}_x, \\ [\mathbf{J}_z, \mathbf{J}_x] &= i\mathbf{J}_y. \end{aligned} \tag{5.2.15}$$

The analysis presented in this section is based on the relation of induced angular momentum to the *eduction* of gauge invariance (see also Ref. 236). One gauge-invariant quantity or observable in one gauge or symmetry can be covariant with another in another gauge or symmetry. The Wu–Yang condition of field overlap, permitting measurement of $\Phi = i \oint \exp \mathbf{A}_\mu d^\mu$, requires coherent overlap. All other effects are either observed at low temperature where thermodynamic conditions provide coherence, or are a self-mapping, which also provides coherence. Thus the question of whether classical $\mathbf{A}_\mu$ wave effects can be observed at long range reduces to the question of how far the coherence of the two fields can be maintained.

Recently, Oh *et al.*[237] have derived the nonrelativistic propagator for the generalized AB effect, which is valid for any gauge group in a general multiply connected manifold, as a gauge artifact in the universal covering space. These authors conclude that (1) if a partial propagator along a multiply connected space ($\boldsymbol{M}$ in the present notation) is lifted to the universal covering space ($\boldsymbol{L}$ in the present notation), i.e. $\boldsymbol{T}_1$: $\boldsymbol{M} \to L$, then (2) for a gauge transformation $\boldsymbol{U}(x)$ of $\mathbf{A}_\mu$ on the covering space $\boldsymbol{L}$, an AB effect will arise if (3) $\boldsymbol{U}(x)$ is not projectable to be a well-defined single-valued gauge transformation on $\boldsymbol{M}$, but (4) $\mathbf{A}_\mu = \boldsymbol{U}(x)\partial_\mu \boldsymbol{U}(x)^{-1}$ (i.e. $\boldsymbol{T}_1 \boldsymbol{T}_2^{-1}$) is nevertheless projectable, i.e. for a $\boldsymbol{T}_2 : \boldsymbol{L} \to \boldsymbol{M}$, in agreement with the analysis presented here. We have stressed, however, that the $\mathbf{A}_\mu = \boldsymbol{T}_1 : \boldsymbol{L} \to \boldsymbol{M}$ have a physical existence, whether the $\boldsymbol{T}_2 : \boldsymbol{L} \to \boldsymbol{M}$ mapping exists or can be performed

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or not. Naturally, if this second mapping is not performed, then no AB effect exists (i.e. no comparative mapping exists).

Although interferometric methods can condition fields into SU(2) or other symmetric form, there is, of course, no control over the space–time metric in which those fields exist. When the conditioned field leaves the interferometer, at time $t = 0$, the field is in exact SU(2) form. At time $t > 0$, the field will depart from SU(2) form in as much as it is scattered or absorbed by the medium.

The gauge invariance of the phase factor requires a multiply connected field. In the case of quantum particles, this would mean wave function overlap of two individual quanta. Classically, however, *every* polarized wave is made us of two polarized vectorial components. Therefore, classically, every polarized wave is a multiply connected field (cf. Ref. 238) in SU(2) form. However, the extension of the Maxwell theory to SU(2) form, i.e. to non-Abelian Maxwell theory, defines mutiply connected local fields in a global covering space, i.e. in a global simply connected form. In the next section, the Maxwell equations redefined in $SU(2)/Z_2$, non-Abelian, or multiply connected form are examined.

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5.3. *Non-Abelian Maxwell equations*

Using Yang–Mills theory,[27] the non-Abelian Maxwell equations, which describe SU(2)-symmetry-conditioned radiation, become Coulomb's law:

$$\text{no existence in SU(2) symmetry;} \tag{5.3.1}$$

Ampère's law:

$$\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} + iq[A_0, \mathbf{E}] - iq(\mathbf{A} \times \mathbf{B} - \mathbf{B} \times \mathbf{A}) = -\mathbf{J}, \tag{5.3.2}$$

the presence of free "magnetic monopoles" (instantons):

$$\nabla \cdot \mathbf{B} + iq(\mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A}) = 0; \tag{5.3.3}$$

Faraday's law:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} + iq[A_0, \mathbf{B}] + iq(\mathbf{A} \times \mathbf{E} - \mathbf{E} \times \mathbf{A}) = 0; \tag{5.3.4}$$

and the current relation

$$\nabla \cdot \mathbf{E} - \mathbf{J}_0 + iq(\mathbf{A} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{A}) = 0. \tag{5.3.5}$$

Coulomb's law [Eq. (5.3.1)] amounts to an imposition of spherical symmetry requirements, as a single isolated source charge permits the choice of charge vector to be arbitrary at every point in space–time. Imposition of this symmetry reduces the non-Abelian Maxwell equations to the same form as conventional electrodynamics, i.e. to Abelian form.

Harmuth's ansatz is the addition of a magnetic current density to Maxwell's equations — an addition which may be set to zero after completion of calculations.[224] With a magnetic current density, Maxwell's equations describe a space–time field of higher order symmetry and consist of invariant physical quantities (e.g. the field $\partial_x \mathbf{F} = J$), magnetic monopole and charge. Harmuth's amended equations are [Ref. 212, Eqs. (4)–(7)]

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + g_e, \tag{5.3.6a}$$

$$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + g_m, \tag{5.3.6b}$$

$$\nabla \cdot \mathbf{D} = \rho_e, \tag{5.3.6c}$$

$$\nabla \cdot \mathbf{B} = \rho_m, \tag{5.3.6d}$$

$$g_e = \sigma \mathbf{E}, \tag{5.3.6e}$$

$$g_m = s\mathbf{H}, \tag{5.3.6f}$$

where g_e, g_m, ρ_e, ρ_m and s are electric current density, magnetic current density, electric charge density, magnetic charge density and magnetic conductivity, respectively.

It should be noted that classical magnetic sources or instanton-like sources are not dismissed by all researchers in classical field theory. For example, Tellegren's formulation for the gyrator[239] and formulations needed in descriptions of chiral media (natural optical activity)[240,241] require magnetic source terms even in the frequency domain. The ferromagnetic aerosol experiments by Mikhailov on the Ehrenhaft effect also imply a magnetic monopole instanton or pseudoparticle

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interpretation,[242–246] the spherical symmetry of the aerosol particles providing SU(2) boundary conditions according to the present view.

Comparing the SU(2) formulation of the Maxwell equations and the Harmuth equations reveals the following identities[224]

U(1) Symmetry:	SU(2) Symmetry:	
$\rho_e = J_0$	$\rho_e = J_0 - iq(\mathbf{A}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{A}) = J_0 + q\mathbf{J}_z$	(5.3.7a)
$\rho_m = 0$	$\rho_m = -iq(\mathbf{A}\cdot\mathbf{B} - \mathbf{B}\cdot\mathbf{A}) = -q\mathbf{J}_y$	(5.3.7b)
$g_e = \mathbf{J}$	$g_e = iq[A_0, \mathbf{E}] - iq(\mathbf{A}\times\mathbf{B} - \mathbf{B}\times\mathbf{A}) + \mathbf{J}$	
	$= iq[A_0, \mathbf{E}] - iq\mathbf{J}_x + \mathbf{J}$	(5.3.7c)
$g_m = 0$	$g_m = iq[A_0, \mathbf{B}] - iq(\mathbf{A}\times\mathbf{E} - \mathbf{E}\times\mathbf{A})$	
	$= iq[A_0, \mathbf{B}] - iq\mathbf{J}_z$	(5.3.7d)
$\sigma = \dfrac{\mathbf{J}}{\mathbf{E}}$	$\sigma = \{iq[A_0, \mathbf{E}] - iq(\mathbf{A}\times\mathbf{B} - \mathbf{B}\times\mathbf{A}) + \mathbf{J}\}/\mathbf{E}$	
	$= \{iq[A_0, \mathbf{E}] - iq\mathbf{J}_x + \mathbf{J}\}/\mathbf{E}$	(5.3.7e)
$s = 0$	$s = \{iq[A_0, \mathbf{B}] - iq(\mathbf{A}\times\mathbf{E} - \mathbf{E}\times\mathbf{A})\}/\mathbf{H}$	
	$= \{iq[A_0, \mathbf{B}] - iq\mathbf{J}_z\}/\mathbf{H}.$	(5.3.7f)

It is well known that only some topological charges are conserved (i.e. are gauge-invariant) after symmetry breaking — electric charge is, magnetic charge is not.[191] Therefore, the Harmuth ansatz of setting magnetic conductivity [and other SU(2) symmetry constructs] to zero on conclusion of signal velocity calculations has a theoretical justification. It is also well known that some physical constructs which exist in both a lower and a higher symmetry form are more easily calculated for the higher symmetry, transforming to the lower symmetry after the calculation is complete. The observables of the electromagnetic field exist in a U(1) symmetry field. Therefore the problem is to relate invariant physical quantities to the variables employed by a particular observer. This means a mapping of space–time vectors into space vectors, i.e. a space–time split.

This mapping is not necessary for solving and analyzing the basic equations. As a rule, it only complicates the equations needlessly.

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Therefore, the appropriate time for a split is usually after the equations have been solved. It is appropriate to mention here the interpretation of the AB effect offered by Bernido and Inomata.[57] These authors point out that a path integral can be explicitly formulated as a sum of partial propagators corresponding to homotopically different paths. In the case of the AB effect, the mathematical object to be computed in this approach is a *propagator* expressed as a path integral in the *covering space* of the background physical space. Therefore, the path dependence of the AB phase factor is wholly of topological origin and the AB problem is reduced to showing that the full propagator can be expressed as a sum of partial propagators belonging to all topological inequivalent paths. The paths are partitioned into their homotopy equivalence classes, Feynman sums over paths in each class giving homotopy propagators and the whole effect of the gauge potential being to multiply these homotopy propagators by different gauge phase factors. However, the relevant point, with respect to the Harmuth ansatz is that the full propagator is expressed in terms of the covering space, rather than the physical space. The homotopy propagators are related to propagators in the universal covering manifold, leading to an expansion of the propagators in terms of eigenfunctions of a Hamiltonian on the covering manifold.

The approach to multiply connected spaces offered by Dowker[247] and Sundrum and Tassic[59] also uses the covering space concept. A multiply connected space, $\boldsymbol{M}$, and a universal covering space, $\boldsymbol{M}^*$, are defined

$$\boldsymbol{M}^* \to \boldsymbol{M} = \frac{\boldsymbol{M}^*}{\boldsymbol{G}}, \tag{5.3.7}$$

where $\boldsymbol{G}$ is a properly continuous, discrete group of isometries of $\boldsymbol{M}^*$, without fixed points, and $\boldsymbol{M}^*$ is simply connected. Each group of $\boldsymbol{M}$ corresponds to n different points $\boldsymbol{qg}$ of $\boldsymbol{M}^*$, where $\boldsymbol{g}$ ranges over the n elements of $\boldsymbol{G}$. $\boldsymbol{M}^*$ is then divided into subsets of a finite number of points or fibers, one fiber corresponding to one point of $\boldsymbol{M}$. $\boldsymbol{M}^*$ is a bundle or fibered space, and $\boldsymbol{\Gamma}$ is the group of the bundle. The major point, in the present instance, is that the propagator is given in terms of

a matrix representation of the covering space $\boldsymbol{M}^*$. Harmuth calculates the propagation in the covering space where the Hamiltonian is self-adjoint. Self-adjointness means that non-Hermitian components are compensated for.[248] Thus, the propagation in the covering space is well defined.

Consequently, Harmuth's ansatz can be interpreted as: (i) a mapping of Maxwell's (U(1) symmetrical) equations into a higher order symmetry field [of SU(2) symmetry] — a symmetry which permits the definition of magnetic monopoles (instantons) and magnetic charge; (ii) solving the equations for propagation velocities; and (iii) mapping the solved equations back into the U(1) symmetrical field (thereby removing the magnetic monopole and charge constructs).

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6. Discussion

The concept of the electromagnetic field was formed by Faraday and set in a mathematical frame by Maxwell to describe electromagnetic effects in a space–time region. It is a concept addressing *local* effects. The Faraday–Maxwell theory, which was founded on the concept of the electrotonic state, *potentially* had the capacity to describe global effects but the manifestation of the electrotonic state, the **A** field, was abandoned in the later interpretation of Maxwell. When, in this interpretation, the theory was refounded on the field concept, and the issue of energy propagation was examined, action-at-a-distance (Newton) was replaced by contact-action (Descartes). That is, a theory (Newton's) accounting for both local and global effects was replaced by a completely local theory (Descartes'). The contemporary local theory can address global effects with the aid of the Lorentz invariance condition, or Lorentz gauge. However, Lorentz invariance is due to a chosen gauge and chosen boundary conditions, and these are not an inevitable consequence of the (interpreted) Maxwell theory, which became a theory of local effects.

According to the conventional viewpoint, the local field strength, $\mathbf{F}_{\mu\nu}$, completely describes electromagnetism. However, owing to the effects discussed here, there is reason to believe that $\mathbf{F}_{\mu\nu}$ does not describe electromagnetism completely. In particular, it does not

describe *global* effects resulting in different histories of local spatiotemporal conditioning of the constituent parts of summed multiple fields.

Weyl[23–26] first proposed that the electromagnetic field can be formulated in terms of an Abelian gauge transformation. But the Abelian gauge only describes local effects. It was Yang and Mills[27] who extended the idea to non-Abelian groups. The concepts of the Abelian electromagnetic field — electric charge, **E** and **H** fields — are explained within the context of the non-Abelian concepts of magnetic charge and monopole. The Yang–Mills theory *is* applicable to both local and global effects.

If the unbroken gauge group is non-Abelian, only some of the topological charges are gauge-invariant. The electric charge is, the magnetic charge is not.[191] That is the reason magnetic sources are not seen in Abelian Maxwell theory which has boundary conditions which do not compactify or reconstitute symmetry and degrees of freedom.

The $\mathbf{A}_\mu$ potentials have an ontology or physical meaning as *local* operators mapping onto *global* spatiotemporal conditions the *local* e.m. fields. This operation is measurable if there is a second comparative mapping of the conditioned local fields in a many-to-one fashion (multiple connection). In the case of a single local (electromagnetic) field, this second mapping is ruled out — but such an isolated local field is only imaginary, because the imposition of boundary conditions implies the existence of separate local conditions and thereby always a global condition. Therefore, practically speaking, the $\mathbf{A}_\mu$ potentials always have a gauge-invariant physical existence. The $\mathbf{A}_\mu$ potential gauge invariance implies the theoretical constructs of a magnetic monopole (instanton) and magnetic charge, but with no singularities. These latter constructs are, however, confined to SU(2) field conditioning, whereas the $\mathbf{A}_\mu$ potentials have an existence in *both* U(1) *and* SU(2) symmetries.

The physical effects of the $\mathbf{A}_\mu$ potentials are observable empirically at the quantum level (effects 1–5) and at the classical level (2, 3 and 6). The Maxwell theory of fields, restricted to a description of *local intensity* fields, and with the SU(2) symmetry broken to U(1), requires no amendment at all. If, however, the intention is to describe

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both local *and* global electromagnetism, then an amended Maxwell theory is required in order to include the local operator field of the $\mathbf{A}_\mu$ potentials, the integration of which describes the phase relations between local intensity fields of different spatiotemporal history after global-to-local mapping.

With only the constitutive relations of e.m. fields to matter defined (and not those of fields to vacuum), contemporary opinion is that the dynamic attribute of force resides in the medium-independent fields, i.e., they are fields of force. As the field–vacuum constitutive relations are lacking, this view can be contested, giving rise to competing accounts of where force resides, such as the opposing view of force not residing in the fields but in the matter (cf. Ref. 249).

The uninterpreted Maxwell, of course, had *two types* of constitutive relations in mind, the second one referring to the energy–medium relation: "…whenever energy is transmitted from one body to another in time, there must be a medium or substance in which the energy exists after it leaves one body and before it reaches the other…" (Maxwell,[8] vol. II, p. 493).

After removal of the medium from consideration, only one constitutive relation remained and the fields have continued to exist as the classical limit of quantum-mechanical exchange particles. However, that cannot be a true existence for the classical force fields because those quantum-mechanical particles are in units of action, not force.

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